Taming errors... pt. 11: How to be precisely imprecise?

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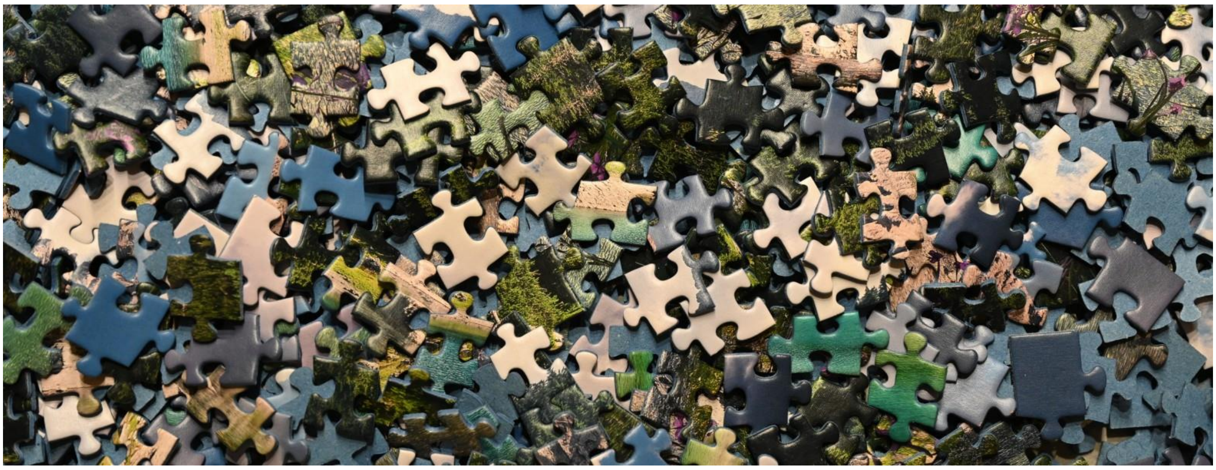

Let's start this issue off with a little quiz which is again orbiting around the subject of accuracy. It is based on a practical dataset of a tunnel. The tunnel itself is 190 metres long, was captured by 45 static laser scans and supported by 61 tacheometric control points (Data courtesy of Rapp Infra AG; thanks to Sebastian Aust). Three different configurations of the network were computed where more and more control points have been removed to simulate more economic scenarios. The tricky question is, which network is the most accurate? Below you will find the mean 3D-residuals among tacheometric control points and registered local target points.

- Network a: 1.5 mm
- Network b: 3.4 mm
- Network c: 2.9 mm

And - what was your choice? If it was the first network, then it was definitely the worst possible choice despite the fact that it features the lowest residuals to control. This becomes a lot clearer once you have a look WHERE the points are located – namely on the outer left, as depicted in Figure 1. Scan stations are represented by large circles, local control points by small circles and ground control points by triangles. In this case all control points were situated on one locally restricted end of the network which yields in huge extrapolation effects.

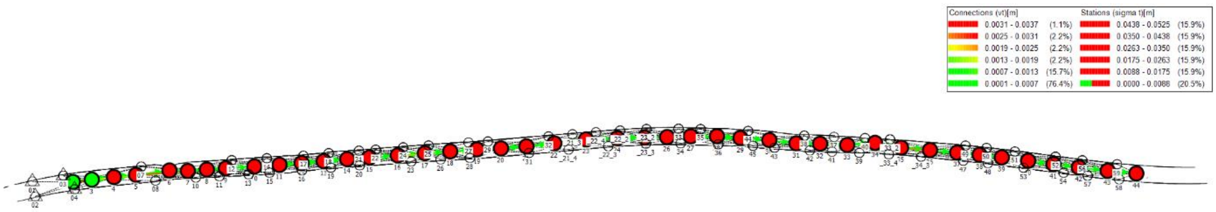

Figure 1: Network configuration a, aka as the whip, features huge extrapolation effects

What I did not mention yet was that the client requested 3 mm (1 Sigma) of global accuracy. If you are now starting to get slightly irritated, then this may be because of the previously mentioned phrases "…*huge extrapolation effects*…", and "…*mean residuals to control = 1.5 mm*…". At first, you may believe that the global accuracy of 3 mm is already satisfied since the misclosures to control are a lot smaller on average than 3 mm. And then there is another schrapnel of words ringing in your head which meant something like "*…the control points were poorly placed*". Then you look again at the mean residuals…still 1.5 mm. Then you remember a key aspect of Taming Errors 10 - residuals to geodetic control points are not telling you the whole story about the quality of a geodetic network. To sum it up, residuals cannot describe the accumulation of errors throughout a network.

Hence, the question is, which expressive quality measure can we use to report the global accuracy for every single scan. The answer is the so called stationing uncertainty. It reports how much a single scan "wiggles" with respect to a given reference. A reference can for instance be estabilished by a single scan, a model or ground control points. Figure 2 features a picture that was already used in the first issue of this series. It contains a "*reference scan*" on the left that is represented by my thumb. It also shows that relative quality measures in the form of residuals to e.g. ground control points or adjancent "*scans*" only allow to draw local conclusions. Residuals are not capable to show how the uncertainty accumulates within a network. In contrast to residuals the stationing uncertainty is a statistical value that is influenced by many factors such as the quality of a scanner's calibration and the entire network configuration. The latter means that it not only considers e.g. every single pairwise registration between scans, inclinometer readings and ground control points but also inherently reflects the impact provoked by their individual location and geometric quality.

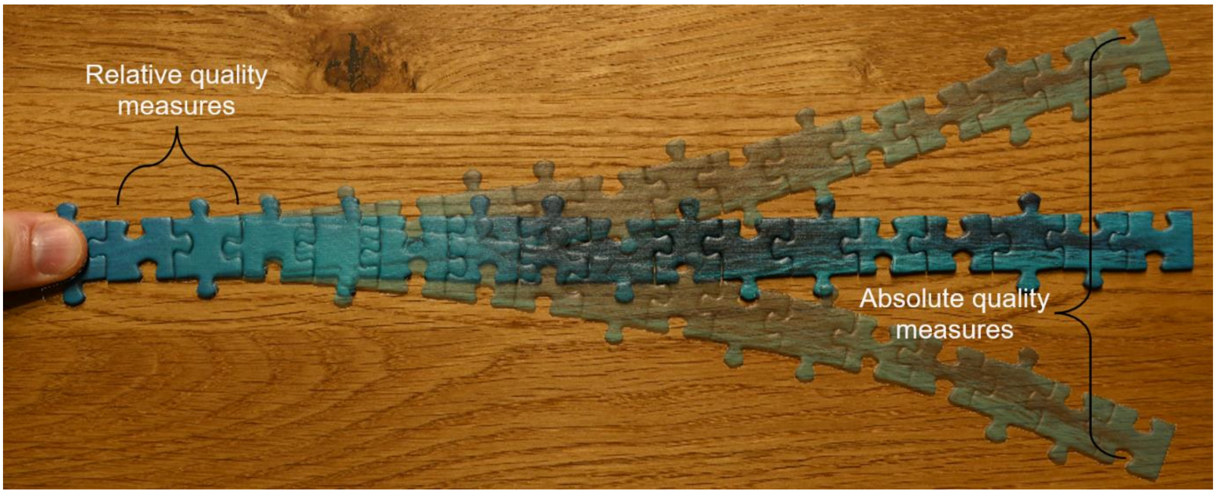

Figure 2: Illustration of relative and absolute quality measures

In order to compute quality measures such as the stationing uncertainty you need two ingredients: a) observations of any kind and b) so called stochastics which reflect the precision of individual observations. These ingredients are then cooked following a mathematical recipe, which is also referred to as the functional model, in order to receive the desired parameters as well as their quality measures.

For the sake of exemplification let’s have a look at how the uncertainty of the z-component of a 3D-point can be computed for tacheometric observations. Tacheometres capture ranges and angles / directions (the difference between two directions is an angle) whose accuracy is reported in spec sheets. Let us assume an angular accuracy of 5” (which equates to 0.0014°) and a range accuracy Sd of 2 mm as well as a zenith distance alpha of 83° and a slope distance of 124 metres. The functional relation to compute the vertical component of a 3D-point follows Sz = sin (alpha) x Sd. Based on this functional model one can compute its uncertainty by using variance-covariance propagation (Ghilani 2019, pp. 99). Using the aforementioned observations and their stochastic values the uncertainty of a 3D-point’s z-component is 3 mm (1 Sigma). In essence, we can do the very same for registrations which will be discussed in the next episode.

If you look carefully at Figure 1 then you’ll notice that only two stations are coloured in green while the rest is tinted in red. Red means that the stationing uncertainty exceeds 3 mm (1 Sigma), green that it satifies the required quality. The highest stationing uncertainty can be found on the outer right of the network and sums up to 52.5 mm – not even close! The accumulation becomes even clearer once you look at Figure 3 where the same outcome was colourised gradually.

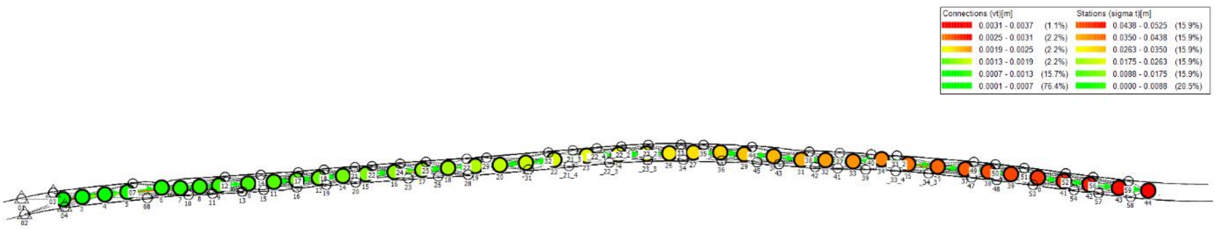

Figure 3: Same network as in Figure 1 – but different colour scale

In order to stabilise the given network, you could simply add four more control points to the other end of the tunnel so that it complies with the configuration in Figure 4.

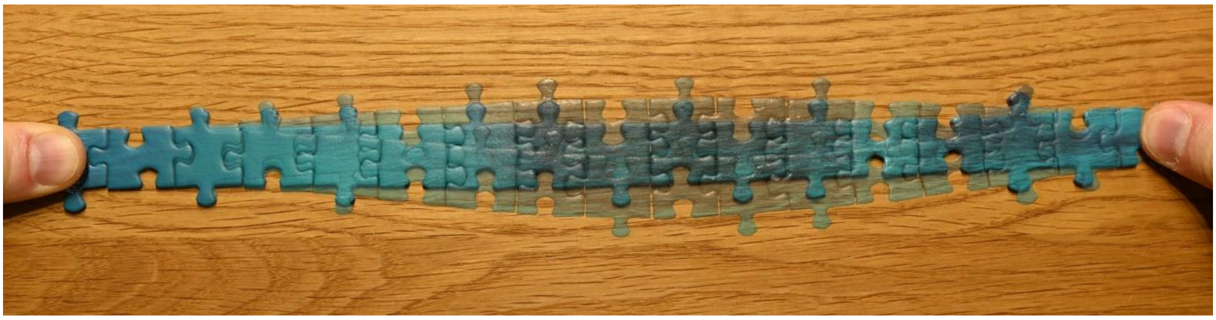

Figure 4: Interpolation between “control points”

Did that help? Well, “no” and yes. On average the residuals to ground control increase to 3.4 mm which would in practice be considered to be a worse result than the initial configuration. On the other hand, the network’s stability has drastically improved where the worst stationing uncertainty with 4.9 mm is about ten-times better than in network a. Good, but still not enough since there is visible sag towards the middle of the network, as depicted in Figure 5.

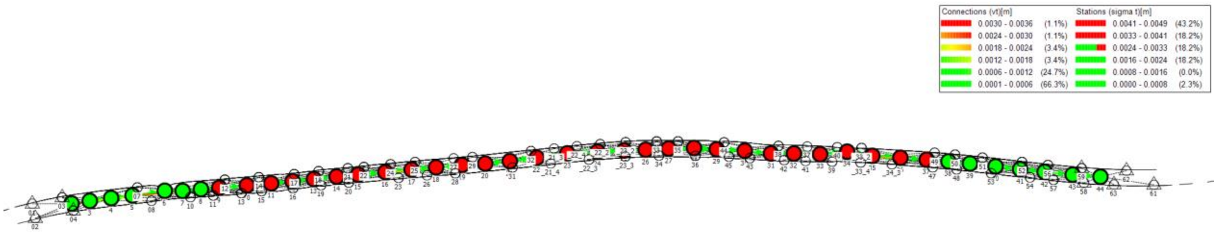

Figure 5: Network configuration b illustrates interpolation effects

In order to stabilise the middle of network b we simply add two more control points close to its centre. On average the residuals to ten tacheometric control in network c sum up to 2.9 mm with a maximum stationing uncertainty of 2.7 mm as depicted in Figure 6. Hence, the specified accuracy has been satisfied and everyone was happy.

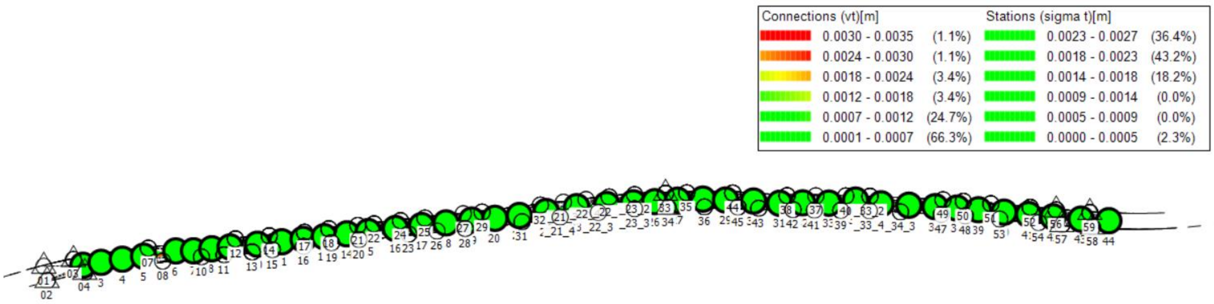

Figure 6: Network c with control points on the outer ends and the middle

References:

Ghilani, C. D. (2017): *Adjustment computations: spatial data analysis*. John Wiley & Sons.

APPENDIX

Since there were a lot of opinions in the laser scanning forum about the influence of inclinometer readings to the network stability, I've added another version of network a without them. Inclinometer readings stabilise the rotation about the x- and y-axes during pairwise registration and consequently tame fluctuations of scan positions in vertical direction. As a result, the worst stationing uncertainty increases from 52.5 mm to 88.6 mm, as illustrated in Figure 7. Ergo: do not play around with this vital piece of information unless you really have to.

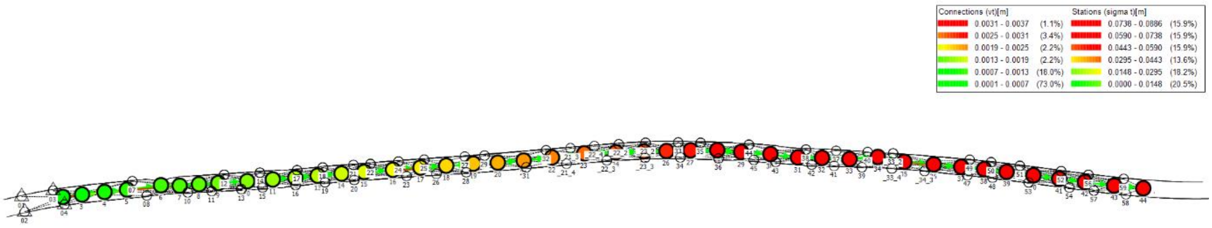

Figure 7: Same control point configuration as in network a – but WITHOUT inclinometer readings

In case you have wondered what could have been achieved without control points at all, below is an example where the reference station was placed in the middle of the network as depicted in Figure 8.

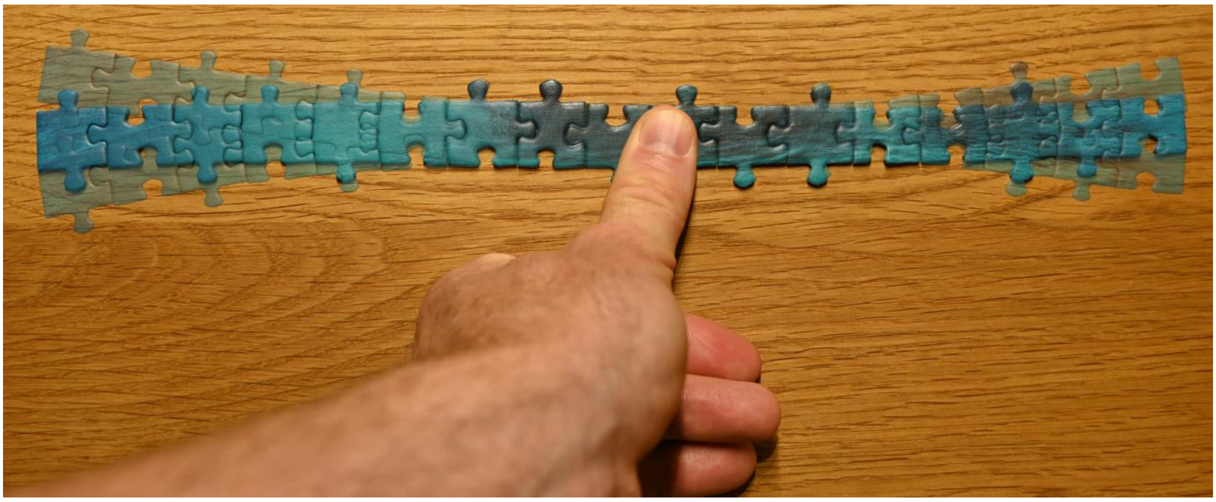

Figure 8: Using a single reference station in the centre of the network

You can clearly see in Figure 9 how the uncertainty increases to the outer ends to a maximum of 9 mm. Since this configuration is essentially a “dead” traverse without any loop closures, well placed control points would have still be required to meet the requirements of the client.

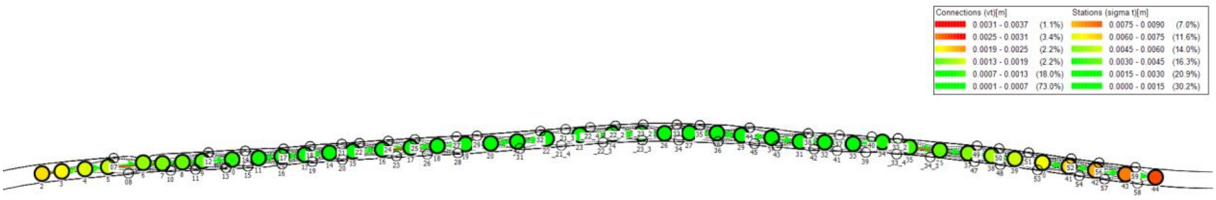

Figure 9: Stationing uncertainty for a network with a single reference station in the middle