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# The calculation of large cable reinforced gas storage systems

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## Abstract

Today, computer models play an important role in the calculation of textile membrane and foil structures. In order to derive high-quality results from a model, the software used must enable a description of a structure that is as accurate and complete as possible. For pneumatically tensioned structures, the creation of the models and the static calculation is a challenge in many cases. A static calculation for membranes and foil structures is geometrically non-linear. The calculation requires the unstressed geometry and the material properties for all elements of the model. For the load case calculation, the external loads and, for pneumatic structures, also the internal pressures or volume data are required. Additional boundary conditions for pneumatic structures are that the loads are deformation-dependent and that the gas law must be considered in certain load cases. If the stresses in the membrane become so big that even the strongest membrane material can no longer bear the stresses, then the membrane must be reinforced with cable nets.

In this paper we show how in our software package the cable net reinforcements can be modelled together with the membrane in one system and then calculated. The cable net can slide on the membrane surface. In this way, it is possible to model the reality accurately.

**Keywords**: Pneumatic systems, gas storage, cable net, reinforcement.

## 1. Introduction

The calculation of pneumatic membrane structures includes form-finding, statics and cutting patterns. This paper deals with cable net reinforced pneumatic membranes, which are indispensable above a certain size in order to keep the membrane stresses within limits; sometimes, for example, also belts are used for gas holders (e.g. Forster B. and Mollaert).

Before we now present the individual steps in the generation of cable-mesh-reinforced pneumatic structures, we will discuss form-finding theory and the statics of pneumatic structures. Subsequently, we will show how arbitrary cable nets can be designed on the pneumatic surface and how the overall system, i.e. membrane and cable-net, is calculated.

## 2. Formfinding for Pneumatics

The theory of the form finding of mechanically and pneumatically membrane or foil structures has its basics in the well-known Force-Density Method (e.g. Stroebel and Holl). By specifying force densities (ratio between force S and stressed length l), the non-linear equilibrium equations become linear and can be solved without specifying initial values.

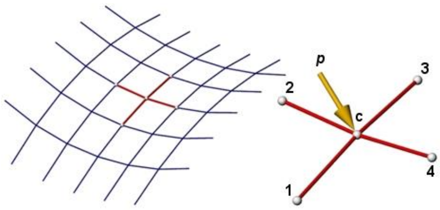



*Figure 1: Four cables in point C*

In the case of a point C connected by 4 cables to fixed points 1,2,3 and 4, the equilibrium conditions are as follows, where the external load vector can be expressed $\boldsymbol{p}^t = (p_x \quad p_y \quad p_z)$ (e.g. Ströbel, D. and Singer, P. et al).

$$\begin{array}{lllllll} (x_c - x_1)\dfrac{S_1}{l_1} + & (x_c - x_2)\dfrac{S_2}{l_2} + & (x_c - x_3)\dfrac{S_3}{l_3} + & (x_c - x_4)\dfrac{S_4}{l_4} & = & p_x \\ (y_c - y_1)\dfrac{S_1}{l_1} + & (y_c - y_2)\dfrac{S_2}{l_2} + & (y_c - y_3)\dfrac{S_3}{l_3} + & (y_c - y_4)\dfrac{S_4}{l_4} & = & p_y \\ (z_c - z_1)\dfrac{S_1}{l_1} + & (z_c - z_2)\dfrac{S_2}{l_2} + & (z_c - z_3)\dfrac{S_3}{l_3} + & (z_c - z_4)\dfrac{S_4}{l_4} & = & p_z \end{array} \tag{1}$$

If one specifies known force densities in (1), e.g. $q_1 = \frac{S_1}{l_1}$, and analogue for $q_2$, $q_3$ and $q_4$, then the equations become linear and result in:

$$\begin{array}{llllll} (x_c - x_1)q_1 + & (x_c - x_2)q_2 + & (x_c - x_3)q_3 + & (x_c - x_4)q_4 & = & p_x \\ (y_c - y_1)q_1 + & (y_c - y_2)q_2 + & (y_c - y_3)q_3 + & (y_c - y_4)q_4 & = & p_y \\ (z_c - z_1)q_1 + & (z_c - z_2)q_2 + & (z_c - z_3)q_3 + & (z_c - z_4)q_4 & = & p_z \end{array} \tag{2}$$

The coordinates of the point $C$ $(x_c, y_c, z_c)$ are the solution of these linear equations. In the following step we want to write the system above by considering m neighbours in the point *C*:

$$\begin{aligned}\sum_{i=1}^{m}(x_i - x_c)q_i - p_x &= 0\\ \sum_{i=1}^{m}(y_i - y_c)q_i - p_y &= 0\\ \sum_{i=1}^{m}(z_i - z_c)q_i - p_z &= 0\end{aligned} \tag{3}$$

The energy which belongs to the system (1) can be written as:

$$\Pi = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) \Rightarrow stat. \tag{4}$$

The internal energy is the expression $\frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v}$. The vector $\boldsymbol{v}^t = (v_x \quad v_y \quad v_z)$ and the matrix $\boldsymbol{R} = diag(q_i \quad q_i \quad q_i)$ show this energy with respect to a single line element $i$ .We can write the inner energy as $\frac{1}{2}q_i(v_x^2 + v_y^2 + v_z^2)$, precisely:

$$\begin{aligned}v_x &= x_i - x_c\\ v_y &= y_i - y_c\\ v_z &= z_i - z_c\end{aligned} \qquad \boldsymbol{R} = \begin{bmatrix} q_i & 0 & 0\\ & q_i & 0\\ sym. & & q_i\end{bmatrix} \tag{5}$$

The chamber of a pneumatic structure has a volume *V*, which is made by an internal pressure $p_i$. The product from internal pressure and volume is a part of the total energy $\Pi$: a given volume $V_0$ leads directly to a specific internal pressure $p_i$: hence the total energy for the form-finding of a pneumatic chamber is

$$\Pi = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) - p_i(V - V_0) \Rightarrow stat. \tag{6}$$

The derivation of the total energy to the unknown coordinates and to the unknown internal pressure ends up with

$$\begin{aligned}
\frac{\partial \Pi}{\partial x} &= \sum_{i=1}^{m}(x_i - x_c)q_i - p_x - p_i\frac{\partial V}{\partial x} &&= 0\\
\frac{\partial \Pi}{\partial y} &= \sum_{i=1}^{m}(y_i - y_c)q_i - p_y - p_i\frac{\partial V}{\partial y} &&= 0\\
\frac{\partial \Pi}{\partial z} &= \sum_{i=1}^{m}(z_i - z_c)q_i - p_z - p_i\frac{\partial V}{\partial z} &&= 0\\
\frac{\partial \Pi}{\partial p_i} &= V - V_0 &&= 0
\end{aligned} \tag{7}$$

In the system (6) the internal pressure $p_i$ can be seen as a so-called Lagrange multiplier. The fourth row in (7) shows, that our boundary condition $V = V_0$ is obtained by the derivation of the energy to this Lagrange multiplier. The vector $(\frac{\partial V}{\partial x} \quad \frac{\partial V}{\partial y} \quad \frac{\partial V}{\partial z})$ describes the normal direction in the point (x, y, z) and the size is the according area. By a set of given force-densities for all elements and a given volume $V_0$ we end up with a pre-stressed and of course balanced pneumatic system with a volume $V_0$ and an internal pressure $p_i$.

## 3. Statics for Pneumatics

By introducing the constitutive equations for the membrane elements into the system (1), we extend the form-finding theory. Now the force-densities $q$ from the form-finding are unknowns and they belong to the material equations.

$$\begin{bmatrix}\sigma_u\\ \sigma_u\\ \tau\end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 0\\ & m_{22} & 0\\ sym. & & m_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_u\\ \varepsilon_v\\ \Delta\gamma\end{bmatrix} \tag{8}$$

We must consider that the membrane axial-stress in *u*- or *v*- direction can be expressed as $\sigma_u = \frac{S_u}{b_u}$ and $\sigma_v = \frac{S_v}{b_v}$. $b_u$ and $b_v$ are the widths of the *u*- and *v*-lines. The force-densities $q$ can be introduced now as: $S_u = q_u l_u$ and $S_v = q_v l_v$. The strains in *u*- and *v*-direction can be written as follows: $\varepsilon_u = \frac{l_u - l_{u0}}{l_{u0}}$ and $\varepsilon_v = \frac{l_v - l_{v0}}{l_{v0}}$ . The angle difference $\Delta\gamma = \gamma - \gamma_0$ is needed for the shear-stress calculation. $\gamma$ is the angle between *u* and *v*-direction; $\gamma_0$refers to the 'non-deformed start-situation' without any shear-stress. The geometrical compatibility must be considered as follows: $l_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2}$ and $\gamma = arccos(\frac{l_u * l_v}{l_u l_v})$, in which $(l_u * l_v)$ means the inner (scalar-) product between *u* and *v*-direction. The shear-stress calculation is guaranteed also for a continuous membrane by the fact that the shear angle is between the non-deformed *u*- and *v*-direction of the material (e.g. Stroebel and Holl).

As already mentioned, additional boundary conditions must be fulfilled for pneumatic structures:

1. The internal pressure loads are deformation dependent.
   These loads are non-conservative. To get correct results software packages should consider these effects, especially also for wind loads.
2. Gas laws must be considered in certain load cases. For static calculations we recommend 4 calculation modes:
   a) Given internal pressure p (snow)
   b) Given volume V (water)
   c) Given product $p{\cdot}V$ (Boyle-Mariotte, for example wind, p as absolute pressure)
   d) Given product $\frac{p{\cdot}V}{T}$ (General gas equation, consideration of temperature, p as absolute pressure)

Mode c (consideration of gas-laws) enables the realistic behaviour of the internal pressure. This mode is important in case of e.g. fast wind gusts. Here the pump systems cannot update the inner pressure in the short time. We can see it as a closed system and by considering the temperature as constant we get the gas law of Boyle and Mariotte $p{\cdot}V{=}const$ in this case. Only if the gas law is fulfilled the membrane stresses get the correct size.

$$
\begin{aligned}
\frac{\partial \Pi}{\partial x} &= \frac{1}{2}\frac{\partial (v^t R v)}{\partial x} - p_x - \frac{\partial V}{\partial x} p_i &&= 0\\
\frac{\partial \Pi}{\partial y} &= \frac{1}{2}\frac{\partial (v^t R v)}{\partial y} - p_y - \frac{\partial V}{\partial y} p_i &&= 0\\
\frac{\partial \Pi}{\partial z} &= \frac{1}{2}\frac{\partial (v^t R v)}{\partial z} - p_z - \frac{\partial V}{\partial z} p_i &&= 0\\
\frac{\partial \Pi}{\partial p_i} &= V - V_0 &&= 0
\end{aligned}
\tag{9}
$$

Equation (9) refers to mode c, here the constant value $(p_{abs}{\cdot}V)_0$ is the given product and row 4 of (9) must be fulfilled in iterations where the unknown internal pressure $p_i$ is adapted.

## 4. Geodesic Lines and Slip Cables

Geodesic lines are solutions of a second order ordinary differential equation. In this paper, however, we will use some other definitions of the geodesic line, which show clearly that a geodesic line corresponds to a weightless prestressed cable stretched frictionlessly over a surface. A geodesic is a line whose geodesic curvature vector vanishes. This is only the case if the plane - created by the tangential and normal vector of the curve - is also the normal plane of the surface, i.e. the normal vectors of the curve and the surface normal vectors coincide at every point of the geodesic line. A prestressed cable on a surface can only be in equilibrium if it is normal, i.e. perpendicular, to the surface. Therefore, the equilibrium position of a prestressed (weightless) cable on a surface is a geodesic line and we define it as slip-cable.

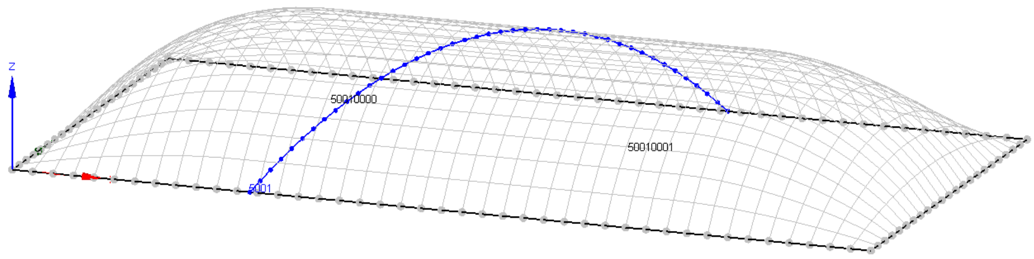



*Figure 2: Geodesic/slip line over surface*

We introduce now the slip-cables in the form-finding stage. Here we define a specific force $F$ within the blue cable. Furthermore we assume the number of the blue cable pieces from this single slip cable to be $n$, the stressed length in a cable to be $l_i$ and the forcedensity to be $q_i$. So we have to add the following lines to equations (7).

$$q_i - \frac{F}{l_i} = 0, \qquad i = 1, n \tag{10}$$

In case of static calculation we assume to have the stiffness of the slip cable (EA) and the sum of all stressed lengths $L$ and unstressed lengths $L_0$.

$$L = \sum_{i=1}^{n} l_i$$
$$L_0 = \sum_{i=1}^{n} l_{0_i} \tag{11}$$

Now the slip cable force densities $q_i$ can be calculated as:

$$q_i = EA \frac{L - L_0}{L_0 \cdot l_i} \tag{12}$$

As $F = EA \frac{L-L_0}{L_0}$ we end up with the same force in all cable pieces.

The green cable (all points on it are fixed) in Figure 3 has the black vectors as reaction forces. The yellow lines show the normal vectors of the surface. The black vectors and the yellow lines are parallel. This means that the green cable is a geodesic line. You can also see in the top view the S-line, which is always obtained as a geodesic line in the case of a cylindrical surface.

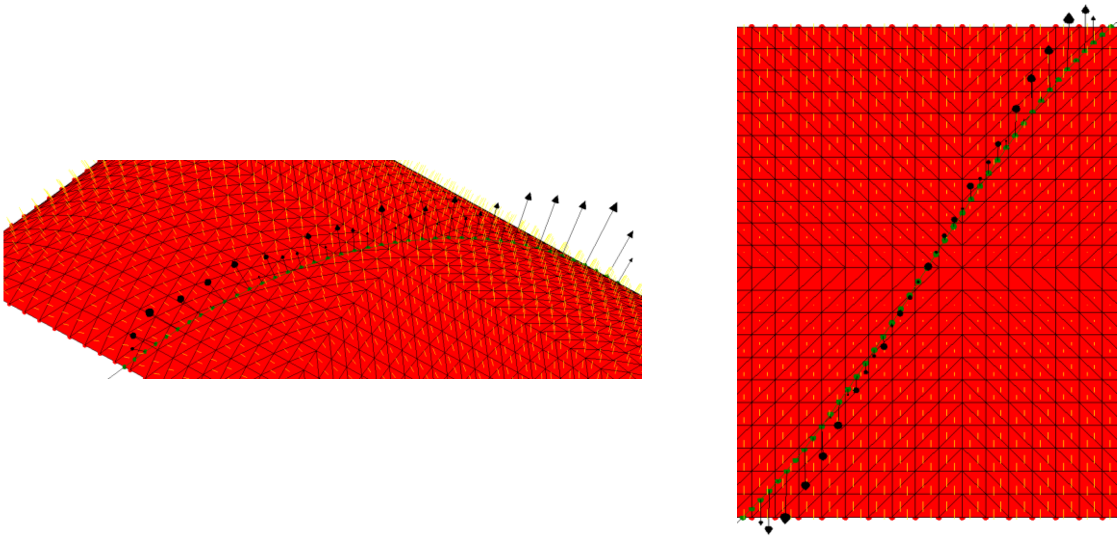
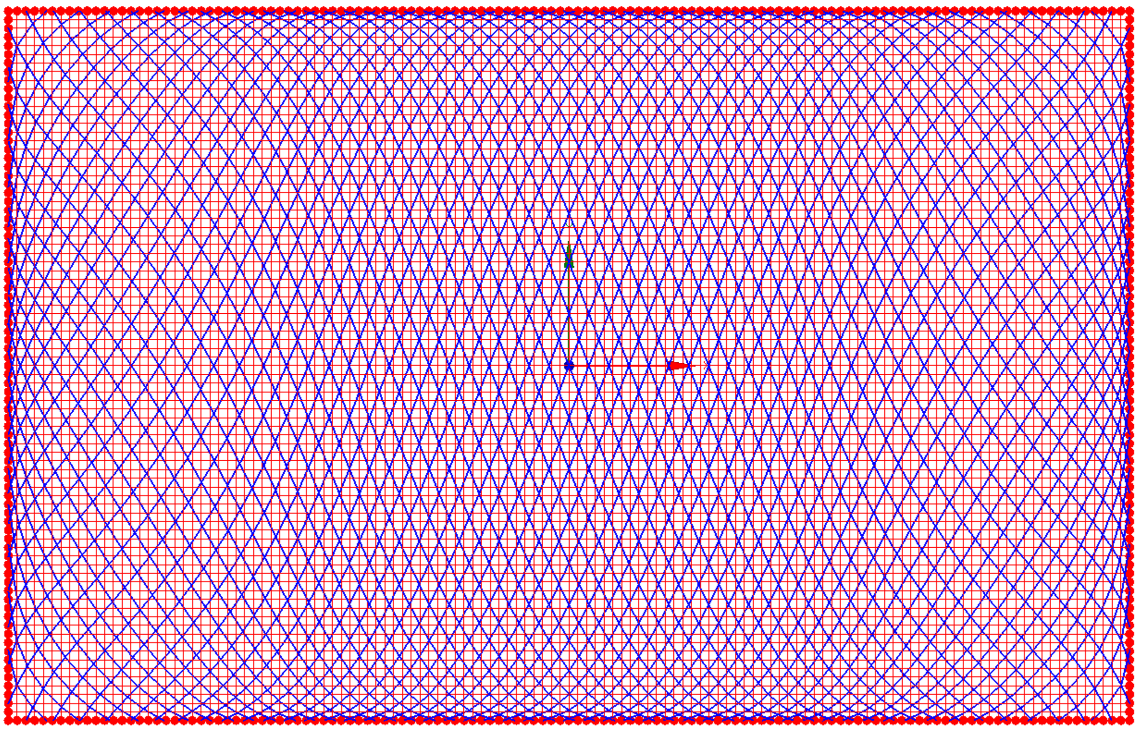


*Figure 3: Slip line over cylindrical surface (side view left, plan view right)*

This means for the calculation of pretensioned (weightless) cables on a pneumatic membrane that these cables are provided with a pretension and they slide friction-free on the surface until they reach an equilibrium position. The condition is therefore simply constant force and the surface forms the support.

## 5. Examples

Our software gives several possibilities to put cables or a cable onto a pneumatic surface. In the following example we generated an equidistant mesh on the surface.



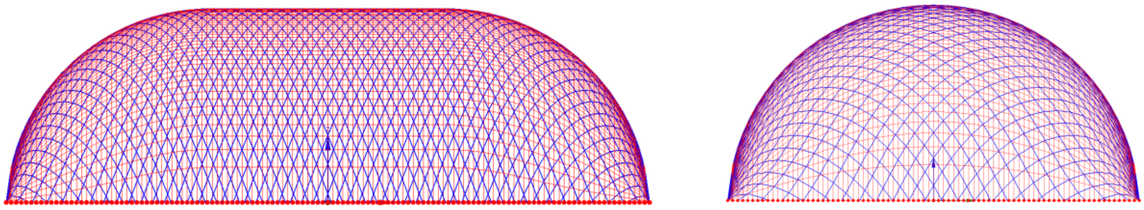


*Figure 4: Big gas holder with equidistant cable mesh*

The following picture shows that the cables constrict the membrane already in the load case of internal operating pressure.

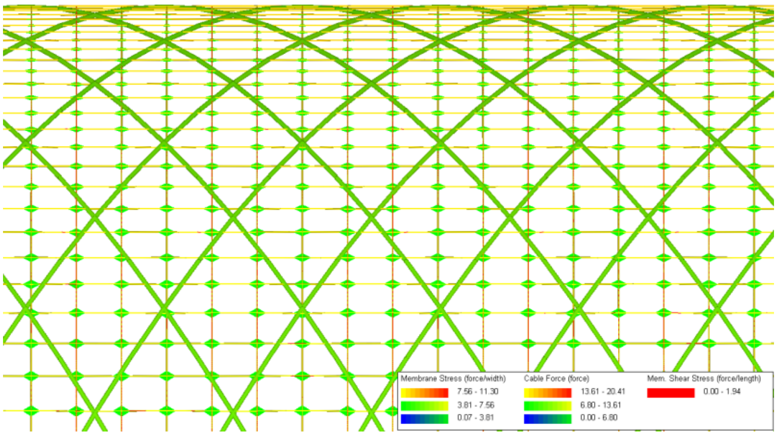



*Figure 5: Constriction of the membrane by cables*

When calculating pneumatic cable net reinforced membranes, the wind load cases are important and relevant for dimensioning.

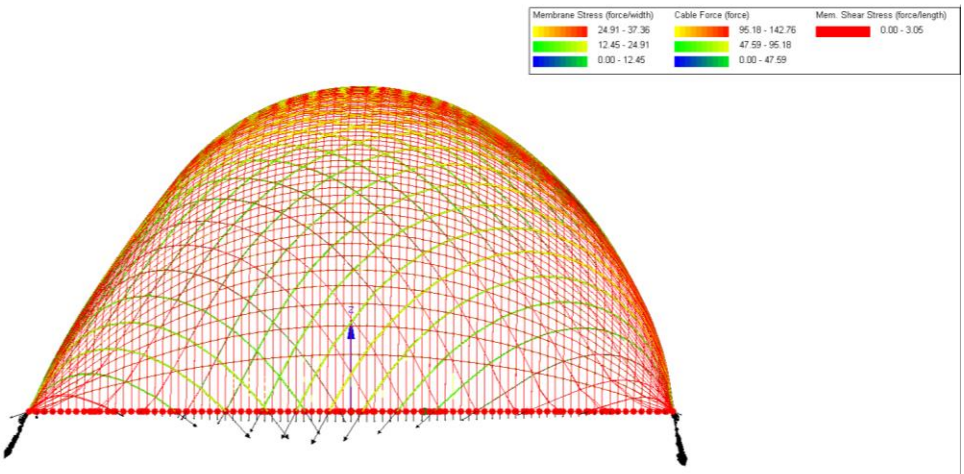



*Figure 6: Deflected form under side wind*

Snow loads should also be considered.

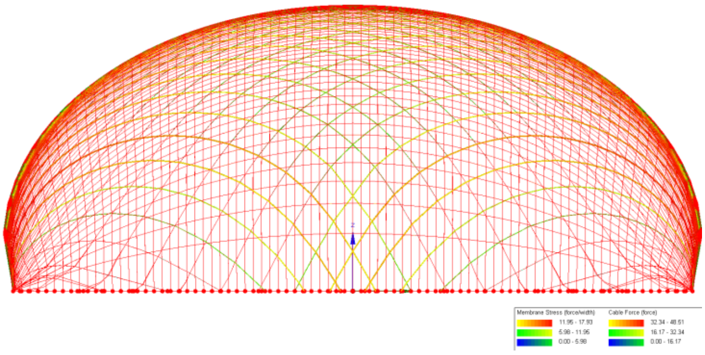


*Figure 7: Deflected form under snow load*

## 6. Conclusion

The calculation methods for form-finding and statics of pneumatic constructions are extended with the help of additional conditions for the combined calculation of cable or cable net reinforced pneumatic membrane constructions. These extended methods are built into the calculation programs. They lead to realistic results, as can be seen from many examples. Even the largest projects can be determined in acceptable calculation times.

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