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HIGH-PERFORMANCE CUTTING PATTERN GENERATION OF ARCHITECTURAL TEXTILE STRUCTURES

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Abstract. *This paper deals with the task of generating high-quality planar cutting patterns for stressed membrane surface structures. Following a brief introduction to the general field of cutting pattern generation, the practical constraints which influence membrane surface structures are presented. The planar sub-surface regeneration strategy used by the Easy design system's state-of-the-art cutting pattern generation tools is described in detail. Particular emphasis is addressed to the use of appropriate modeling strategies for dealing with the various complex materials used in practical structures. These range from very flexible uncoated fabrics though conventional coated polyesters to the high-stiffness glass textiles. The paper concludes with an example which illustrates the capabilities of Easy´s cutting pattern generation tools.*

1 Introduction

1.1 The Membrane Structure Design Process

This paper is concerned with the design of architectural membrane surface structures fabricated from planar strips of either textile fabric or plastic film. The three main processes involved in the design of membranes for architectural structures are; *Form-finding*, *Statical Load Analysis* and *Cutting Pattern Generation*. *Form-finding* is the name given to the problem of determining a structural form, in most cases a surface, which is in force equilibrium and satisfies additional design constraints. *Statical Load Analysis* must typically be performed using geometrically non-linear structural analysis software in order to check that the form-found surface satisfies ultimate and serviceability constraints. Finally, the formfound surface must be converted into a set of planar cloths for fabrication, this is termed *Cutting Pattern Generation*. Consideration of *Cutting Pattern Generation* deals with the problems of defining cloth subdivisions of large surfaces, and ensuring that these sub-surfaces are two-dimensionally developable.

Originally this problem was addressed through the construction of physical models for both *Formfinding* and *Cutting Pattern Generation*. After several decades of continual development, sophisticated software systems are now routinely used during the design process. Today, the main focus for system enhancement lies in the creation of interactive tools to facilitate design optimisation, particularly the control of cutting pattern seam layout. The specification of the actual layout itself is best handled by interactive input from the design engineer. Fig. 1 shows a typical high point surface as formfound, while Fig. 2 shows the locations of the inter-cloth seams during the subsequent cutting pattern generation procedure.

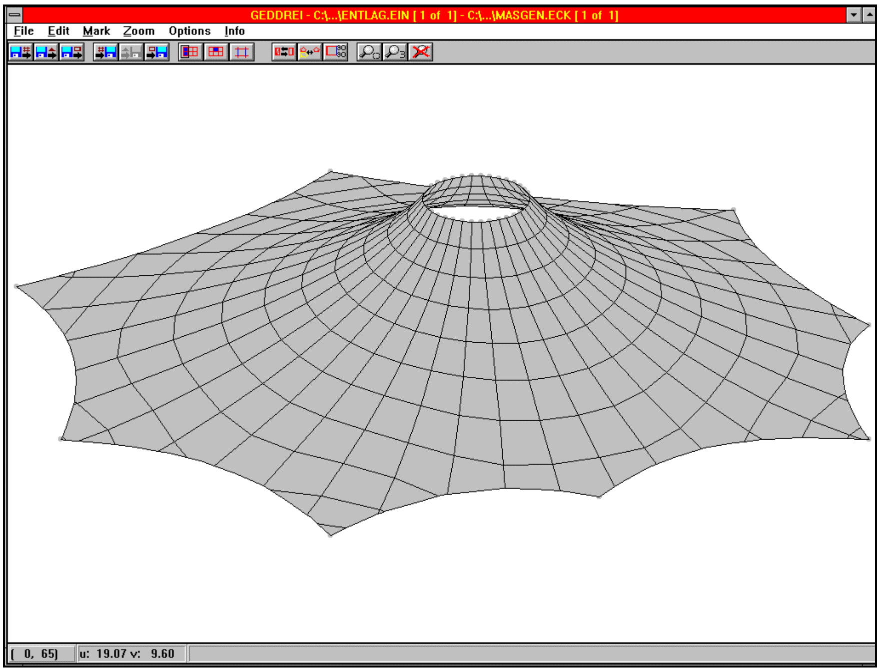

Figure 1: Typical high point surface structure.

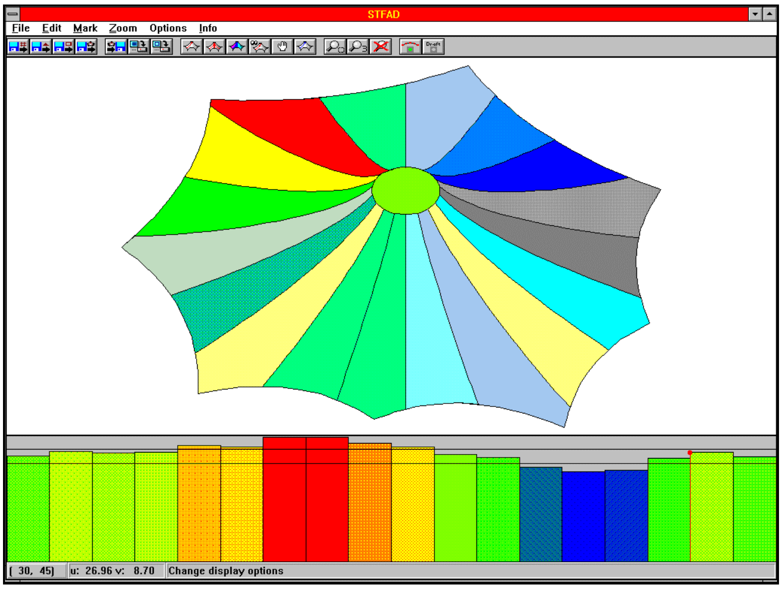

Figure 2: Easy system cutting pattern layout for the high point surface shown in Fig. 1 with graph of cloth widths.

1.2 The Cutting Pattern Problem

At its most general the cutting pattern generation problem may be defined thus. Given a surface *S*, determine a set of *n* planar sub-surfaces $\{s_1, s_2, s_3,..., s_n\}$ such that the distortion between *S* and *S´* is minimised, where *S´* is a surface of type *S* created by reassembling the sub-surfaces. This is illustrated in Figure 3.

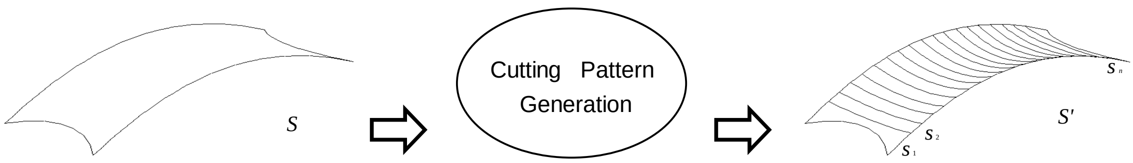

Figure 3: Cutting pattern generation procedure

In the trivial case where *S* is two-dimensionally developable, such distortion can be reduced to zero. Generally, however, the surfaces which are used in practical membrane structure designs need to possess strong double curvature. The main task therefore becomes one of deciding where and how to introduce the minimum distortion necessary. Additionally the specific nature of a particular application area may impose additional constraints. Control of distortion is achieved through orientation and spacing of cloth seam lines. Clearly in areas of particularly high curvature it may be necessary to reduce the cloth widths.

A number of problems are associated with the cutting pattern generation. These are:

- The geometry of the surface is not defined without prestress. The unstressed state has no defined geometrical shape.
- The surfaces can not be described by closed mathematical expressions. They must be described by discrete surface points which must correspond to the state of force equilibrium in the structure.
- The membrane surface is prestressed and doubly curved to resist external loading. It is therefore not developable to the plane without distortion.
- Wrinkles in the stressed surface are to be avoided.
- The pattern precision must be very high in the case of stiff materials such as glass textile.
- The planar strips have to be as straight as possible in order to optimise the pattern from both cloth material wastage and performance viewpoints.
- The material elastic properties of textile are extremely non-linear. Therefore textile structures can not be modelled using a homogeneous stress-strain diagram.
- Seam lines and border reinforcement will additionally complicate the structures elastic response.
- In order to establish the desired working surface pre-stress the patterns must be strain compensated. Deciding how much smaller to make the patterns compared to the stressed geometry is extremely complex
- The manner of sewing the planar strips together as well as the boundary conditions also affect the pattern compensation process.

1.3 Constraints for Membrane Structure Pattern Generation

Cloths must be cut from fabric rolls of relatively narrow width. For economic reasons it is desirable that each cloth should maximise use of the available width. The use of geodesic seam lines are therefore particularly appropriate in almost all cases. It is sometimes economically advantageous for cloths to be patterned with one straight side. Seam lengths of adjacent cloths should be the same, and cloth distortion at structure borders must be avoided.

The coated textiles used for membrane structure projects are unable to withstand prolonged shear loading. Consequently, cloth layout should be arranged such that the principal membrane stresses approximately coincide with the fabric weave directions. From an aesthetic view point it is desirable that consideration is given to the impact of seam lines. Given that the other constraints mentioned above already serve to impose visually acceptable seam orientation, in practice this requirement is largely redundant. However, adjacent cloths should be patterned such that cloth widths vary gradually from area to area. Due to the additional thickness of seams, fabrication is facilitated if four cloth corner junctions are avoided. If such seam layout is required it should be achieved through the use of on-site construction seams. It is generally best if seams are kept clear of boundary corners.

2 Pattern Generation Strategies for Textile Structures

2.1 Modelling of Discrete Spatial Surfaces for Patterning

In order to overcome all the difficulties mentioned above, the geometrical shape of the membrane structure has to be modelled in an appropriate way. This can be achieved by a network of nodes, links, triangles and surface polygons. Through the proper management of these network objects powerful tools may be established for dealing with the flattening process of doubly curved surfaces.

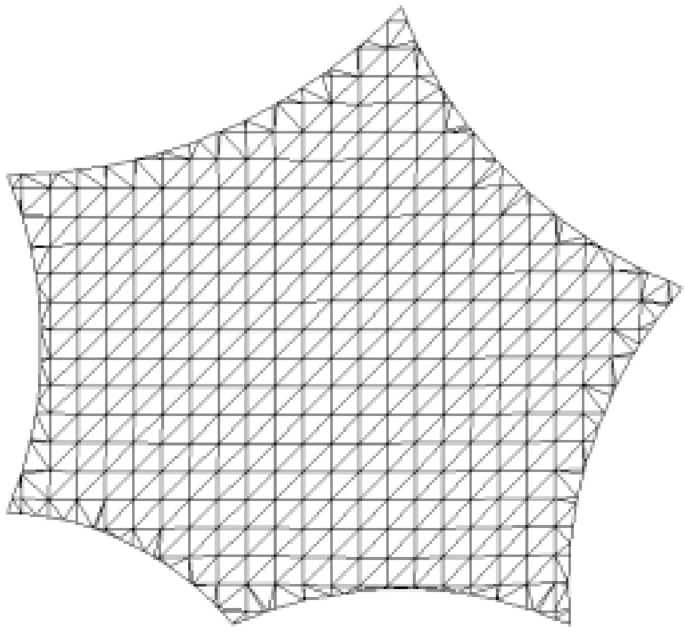

Figure 4: Discrete surface model based on triangles.

Based on Euler's rule the surface polygons provide the necessary information for a unique definition of the necessary geometrical properties of the surface including the normal vectors.

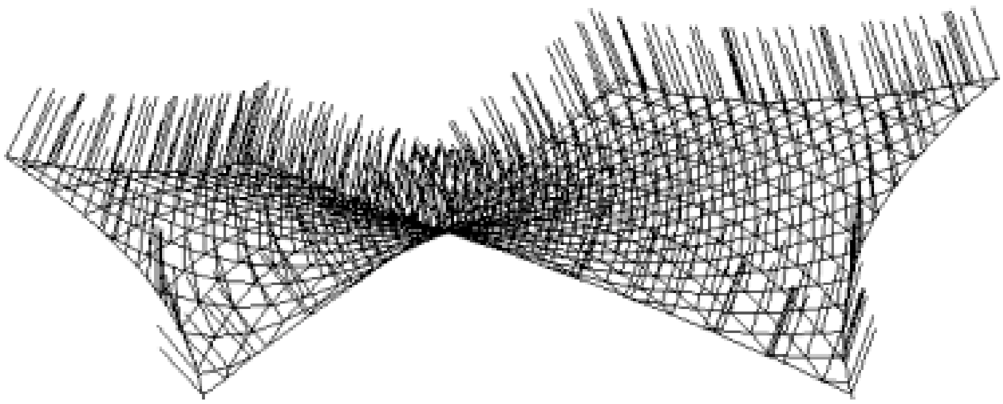

Figure 5: Normal vectors, a prerequisite for describing the curvature of the surface.

Rather than perform surface form-finding and planar cutting pattern generation simultaneously, it is advantageous to separate the tasks and perform each sequentially. The cutting pattern problem may then be solved by generation of inter-cloth seam lines across the form found surface mesh and subsequently cutting the surface into individual sub-meshes according to these lines. The sub-surfaces will generally be non-developable and must therefore be flattened. Such flattening can be performed in a variety of ways. Clearly strategies which seek to minimise overall distortion are best, though it is important to be able to weight adjustment between different parts of the cloth. Generation of strips with one side straight may be achieved by additionally constraining the nodes of the straight side to lie along an axis during the flattening procedure.

2.2 Generation of Geodesic Lines

Geodesic lines across a surface are mathematically defined by differential equations in terms of their direction. At each point they are locally straight relative to the plane tangential to the generation surface. Geodesic lines do not curve in the tangential plane. In addition the shortest path between two points over a surface will be geodesic. It is the fact that a geodesic is straight rather than its length which leads to its excellent suitability for seam generation in textile architectural applications. For an infinitely narrow cloth, the resulting planar pattern from a geodesic generation will have straight sides. As the cloth widths increase, and the corresponding patterning distortion increases, the sides will curve. Consequently, a surface properly patterned on the basis of geodesic lines can have cloths which minimise cloth usage as well as the angles between textile weave and surface principle stresses.

As with the theoretical definition, the techniques used for the computational generation of geodesic lines can be broadly divided between *Length* and *Angular* based approaches. With the former the position of the intermediate points are adjusted such that the total length of the line is minimised. This can be achieved in a number of ways. Seam links with very high tension may be introduced into the formfinding mesh and used to pull the seams into geodesic paths. Clearly, so as not to affect the formfound surface too much, the out of plane components of these high string tensions must be suppressed together with the end point reactions. Alternatively, such strings may be slid about over a fixed surface using a length minimisation cost function.

A geodesic line across a discrete surface can be represented as a polyline in terms of the intersection points with the sides of the triangles. The sequence of these intersected facets can be developed without distortion. The intersecting polyline will be a straight line between the starting and end point. One simplistic approach to the creation of geodesics over triangulated surfaces is based on this property. By propagating line/triangle intersections from facet to facet geodesic surface partitions can be created. Although effective in some situations, the drawback with such an approach is that numerical errors accumulate across the surface.

The purest approach is to use an algorithm which adjusts the position of the nodes according to observations on the normal plane angles themselves. In such an approach the surface tangential planes in the neighbourhood of each line point are determined in order to define multiple local coordinate systems. By constraining the line nodes to lie on these planes, it is possible to develop linear adjustment problems with observations on the angles and single degrees of freedom for each point. After solving each linear problem, the nodes need to dropped back onto the surface and the process repeated until convergence is achieved. If desired, the spacing between the intermediate nodes can be modified by re-discretisation along the generated line.

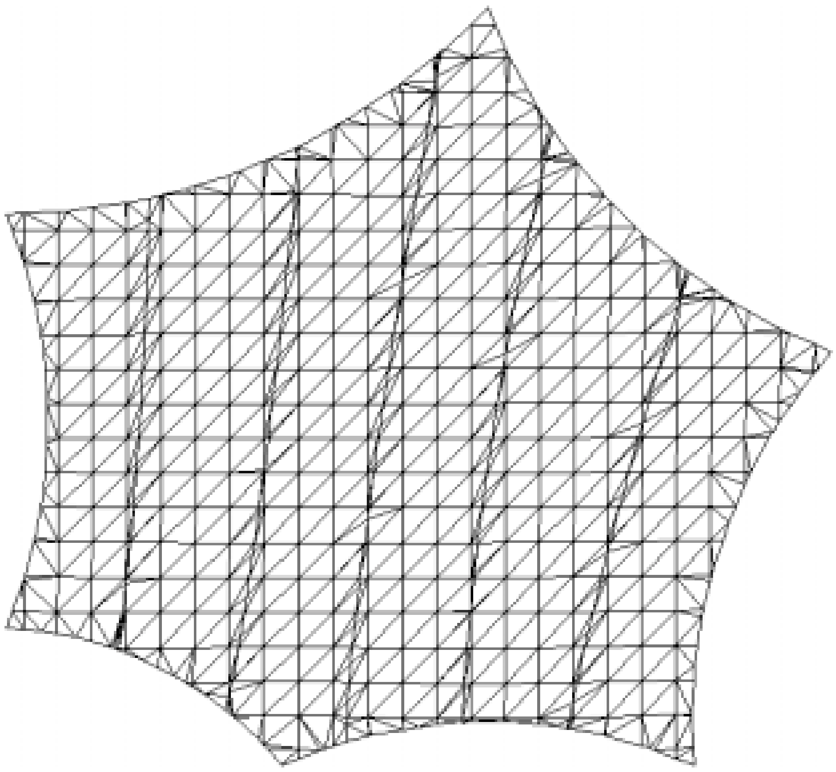

Figure 6: Geodesic lines splitting a discrete membrane surface into strips.

2.3 Flattening of Spatial Surfaces

The flattening of three-dimensional surfaces is a typical map projection transformation task. The flattening process should end in a planar surface with minimal distortion, both global and local.

Using geodesic intersection lines, the discrete surface may be cut into strips. The resulting strips are discrete surfaces which can be modelled favourably by nodes, links, triangles and polygons in the same way as the original structure. Each strip's boundary polygon consists of surface boundary lines and geodesic lines.

For the discrete triangular surface description, the simultaneous mapping of the surface triangles to the planar triangles of the individual strips is the key to the flattening problem.

The mapping of the triangles may be described by affine transformations of the following type, where **x**´ is the affine projection of **x**, the coordinates of the triangle vertices:

$$\mathbf{x}' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x'_0 \\ y'_0 \end{bmatrix} = \mathbf{A}\mathbf{x} + \mathbf{x}'_0 .$$

The affine transformation results in a mapping where parallel straight lines are parallel in the transformed picture and relational distances inside the triangles remain unchanged. The deformation of each triangle will be modelled by three independent parameters. These can be the triangle side length deformations or Green's distortion parameters.

2.4 Strip flattening

For modelling the flattening of strips subject to additional boundary conditions, such as the fixation of border lines to be straight or border line lengths to have prescribed values, additional tools are required. With the **Easy** system the flattening process is based on the normal vectors at each surface point. They define individual projection planes to which the neighbouring points are orthogonally projected. Due to the overlap of the local co-ordinate systems an interconnected transformation problem results which can be solved by a weighted minimisation of the distortions of the co-ordinates following Gauss's method of least squares. The equations read:

$$\mathbf{0} + \mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} t_a & -t_b \\ t_b & t_a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x'_0 \\ y'_0 \end{bmatrix} - \begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T}\mathbf{x} + \mathbf{x}'_0 - \mathbf{x}' .$$

Minimising the weighted sum of $\mathbf{v}^2$ results in a system of linear equations for the planar co-ordinates **x**´, and the transformation parameters of each co-ordinate system. The system of equations may be easily extended enforcing functions of the planar co-ordinates to fulfil additional constraints as mentioned above. As the topological description of the strips remain unchanged, the distortion of each triangle can be evaluated and any additional surface points can be mapped correctly to their planar projection position by applying the affine transformation.

During the flattening the constraint that adjacent cloth seam lengths are identical can be directly enforced. This is possible in the **Easy** system but usually a sequential methodology is used where the enforcement is performed by a separate 2D adjustment. Such separation of tasks enables a closer scrutiny of the progressive build up of patterning distortion.

2.5 Strip Compensation

As the strips will not be cut under pre-stress, a strain compensation of the strips is required. The direction of warp and weft is defined for the flattened strip. Although the compensation could be incorporated into the flattening process it is generally superior to apply the compensation after flattening. This is especially true when the strip seams are reinforced by cables or belts or when they end at fixed boundaries. For those boundaries, the compensation should be close to zero and then gradually increase. The process is extremely complex and can still not be modelled completely computationally satisfactorily due to the absence of material data. Consequently the designer must have complete control over the relative distribution of the degree of compensation.

2.6 Planar Sections versus Geodesic Lines

Some patterning systems still exclusively use lines defined by the intersection of the structural surface and simple planes. The deficiency of such a strategy compared to a geodesic approach has been long established. For example, in Figs. 8 and 9 two patterns are shown for the simple saddle surface shown in Fig. 7. In Fig. 8, vertical planar cut patterning was used. The characteristic ‘banana’ cloths may be compared to those derived from a geodesic patterning which are shown in Fig. 9.

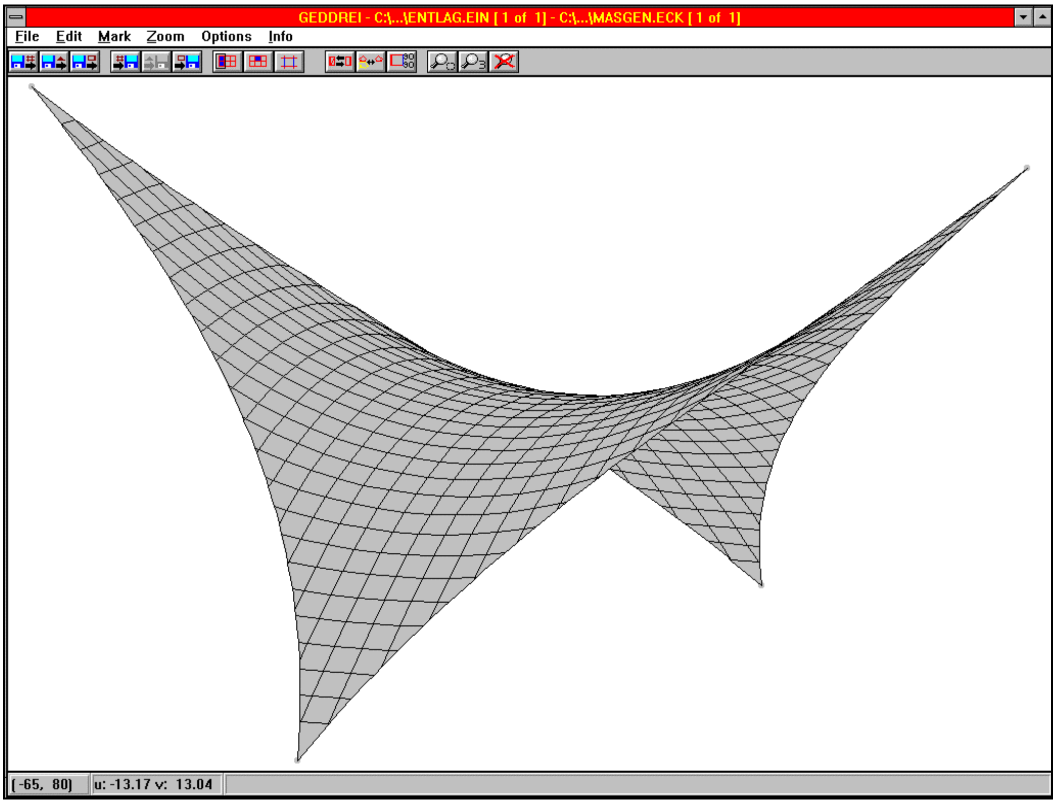

Figure 7: Simple saddle surface.

There are some situations where a planar section is the most appropriate line type. For example, in symmetrical surfaces a vertical cut may be the best choice for the seams, or cloth centres, running along the symmetry axes. Similarly, where roofs must connect to rectilinear supports, in some cases it is better to cut the form-found surface with planar sections, rather than formfinding with rectilinear boundaries.

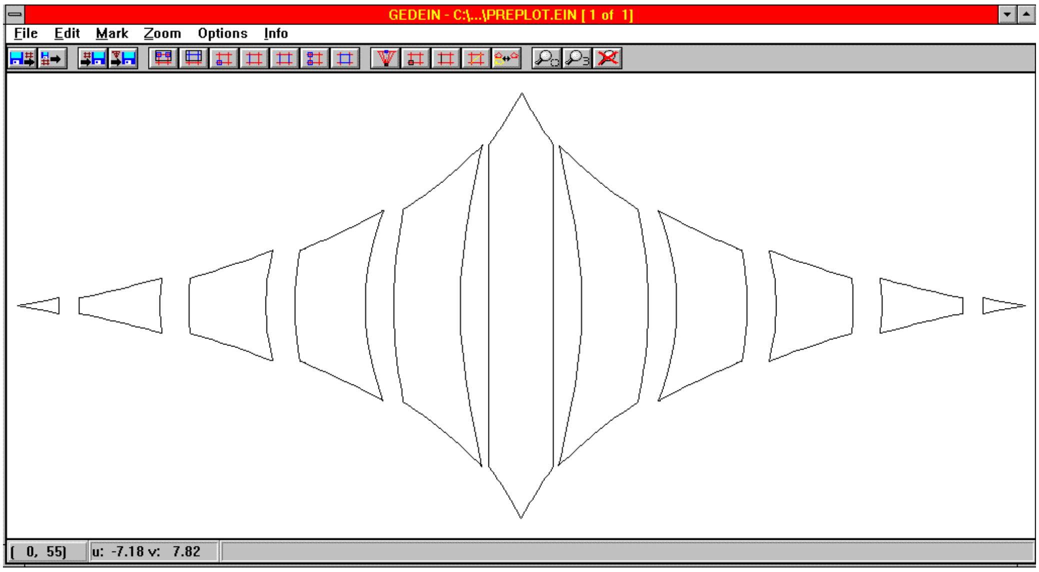

Figure 8: Vertical planar cut patterning of saddle shown in Fig. 7.

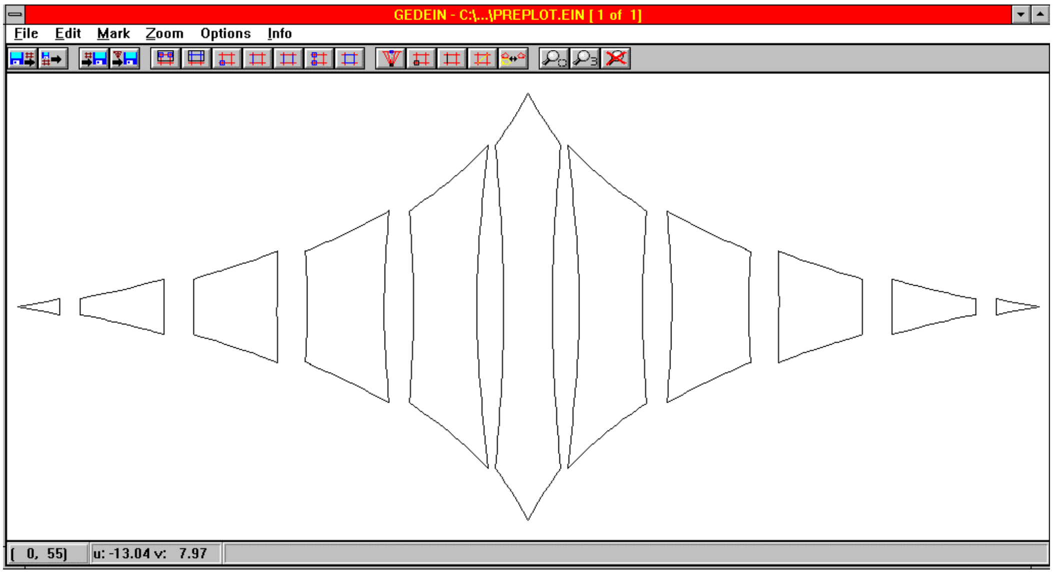

Figure 9: Geodesic patterning of saddle shown in Fig. 7.

2.7 Semi-Geodesic

Whatever their basis, most high quality cutting pattern systems exploit in some way the benefits of *Geodesic* seam line generation. In a small number of situations, however, *Geodesic* lines will not be the most suitable choice. This is particularly the case when patterning surfaces whose curvature does not strongly constrain the *Geodesic*. For example the ends of some airhalls can approach spherical shapes. In such situations it can be difficult to keep cloth centre widths controlled. A pure geodesic line is usually defined in terms of only its end point positions. For better user control, it would be useful to be able to constrain the generated lines to additionally pass through specific interior surface points. Unless such constraining points lie on the pure *Geodesic* path, then the generated line will not be straight relative to the surface. Accordingly the term *Semi-Geodesic* is used to distinguish such lines.

When generating a semi-geodesic line through the introduction of a single additional inner constraint, the resulting line will have constant in-plane curvature. As may be expected the introduction of constant curvature permits the exploitation of a number of efficient strategies. In particular it is possible to apply distributed fictitious loading in the plane of the surface to deflect the curve the required amount.

3 Generation of the Cutting Pattern of the Pilgrim's Tents for Phase II of the Mina Valley Project

3.1 Project Description

Amongst the most interesting of stressed textile projects in recent years has been the Mina valley project in Saudi Arabia. Every year two million followers of the Islamic faith make the Haj pilgrimage to the holy city of Mecca. As part of the event, these pilgrims travel to nearby mount Arafat where they spend the night camping in the Mina valley. Following a series of fatal disasters culminating in a large fire in 1997, the Saudi government decided to replace the existing cotton structures with fireproof teflon coated glass tents. Phase I of the project, which was carried out by *Koch*, *SL* and *Tensys* during 1997 and 1998, provided 25% of the planned 40,000 tents. Phases II and III were carried out by a completely different group with the membrane engineering provided by *technet GmbH*, applying the **Easy** system. 30,300 tents were built and installed during Phases II and III. With a covered area of 1,795,000 m2 this is by far the world's largest lightweight structure project.

As with Phase I, the majority of the tents were to be constructed from six modular designs. These rectilinear structures ranged in size from 4x4m through 8x6m, 8x8m, 8x9m, 8x10m to 8x12m. The general appearance of the tent forms needed to be similar to the original configurations. These were patterned from four triangular panels each of which was fabricated from cloth strips with the textile weft parallel to the outer boundaries. Since there were such a great number of tents to fabricate the issue of cloth wastage was paramount. The textile was procured in a very large number of different widths. Pattern sets were therefore needed for each of these widths and for each of the different roof configurations.

3.2 Cutting Pattern Generation

Although the overall sizes of the various tent designs are quite modest, the technical challenge of patterning the structures is of the highest level. It is a little known paradox of textile structure design that smaller structures are, in general, more difficult to pattern than larger ones. This is due to the fact that with larger structures the limitation of maximum fabric width necessitates the use of many cloths. The 3D to 2D patterning distortion is therefore relatively low for each cloth. With smaller structures the maximum fabric width is relatively large compared to the stuctures overall dimensions. Since cutting, preparing and welding each seam is expensive, it is inevitable that fabricators wish to use the minimum number of cloths in any application. Usually the higher distortion experienced by the relatively wider strips used in smaller structures is mitigated by the fact that lower stiffness membranes are used. In this case, however, the textile used was not only PTFE coated glass, but also a very heavy duty grade. With a completely non-adjustable fixed boundary, the pattern generation problem therefore becomes extremely sensitive.

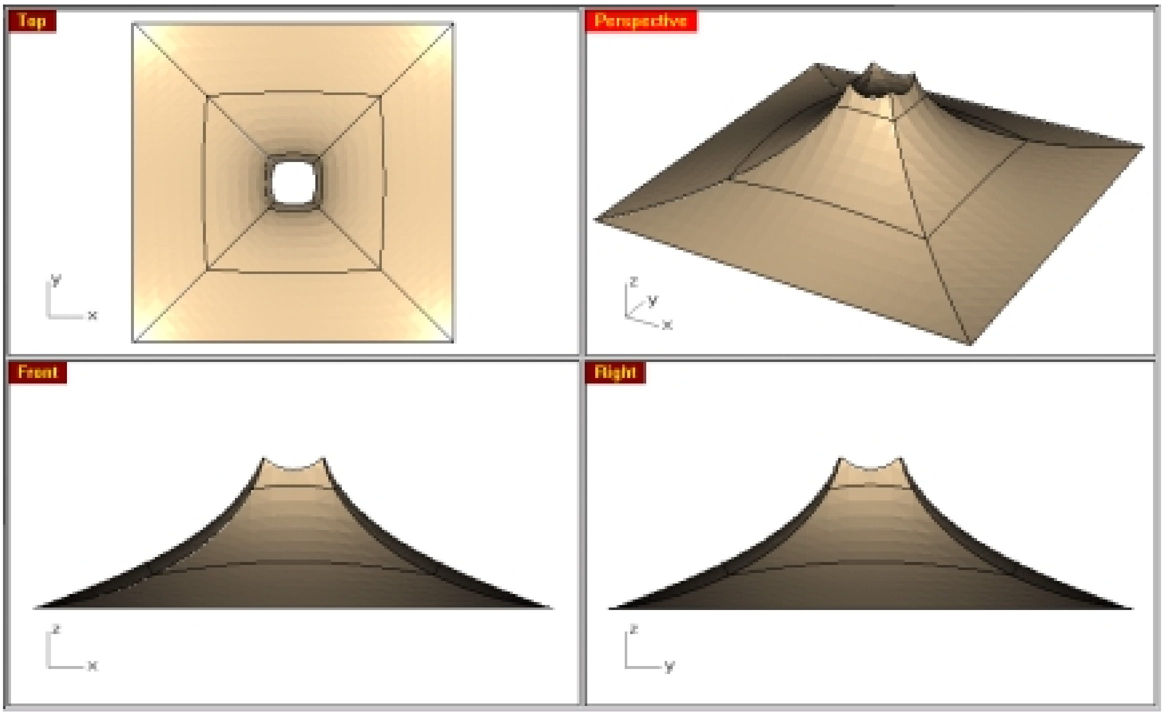

Figure 10: 8x8 production surface showing the new seam layout.

For the Phase I roofs the cloths were oriented in the same way as the original cotton tents, namely with the weft parallel to the outer boundaries. This is good engineering practice when the roof is subject to high downward loads such as snow. The reason for this is that the seams do not experience the maximum fabric stress, and since the warp is stiffer less deflection will be experienced. The higher stiffness of the warp does however lead to problems for this configuration. In order to achieve the desired constant stress, the cloth compensation must be much higher in the weft direction. This means that the fabric needs to be strained more at the borders during installation than if the warp runs parallel to the frames. More serious is the very low strain range which is permissible in the fabric during vertical adjustment of the mast top. The practical result of this is that tents patterned according to the Phase I layout are fundamentally more susceptible to wrinkling than those patterned with the warp parallel to the frame boundaries. Since wrinkling was a major problem with the Phase I roofs it was decided to change the layout direction for Phase II. Due to the very high strength of the glass textile used, resistance to applied load was not critical.

The cutting pattern generation was performed using **EasyCut**. The first stage in the cutting pattern generation procedure was the generation of the geodesic lines. Fig. 10 shows one of the seam layouts for the 8x8 roof. Fig. 11 shows the result of cutting up the form-found surface according to these seam lines together with additional geodesic cloth centre lines.

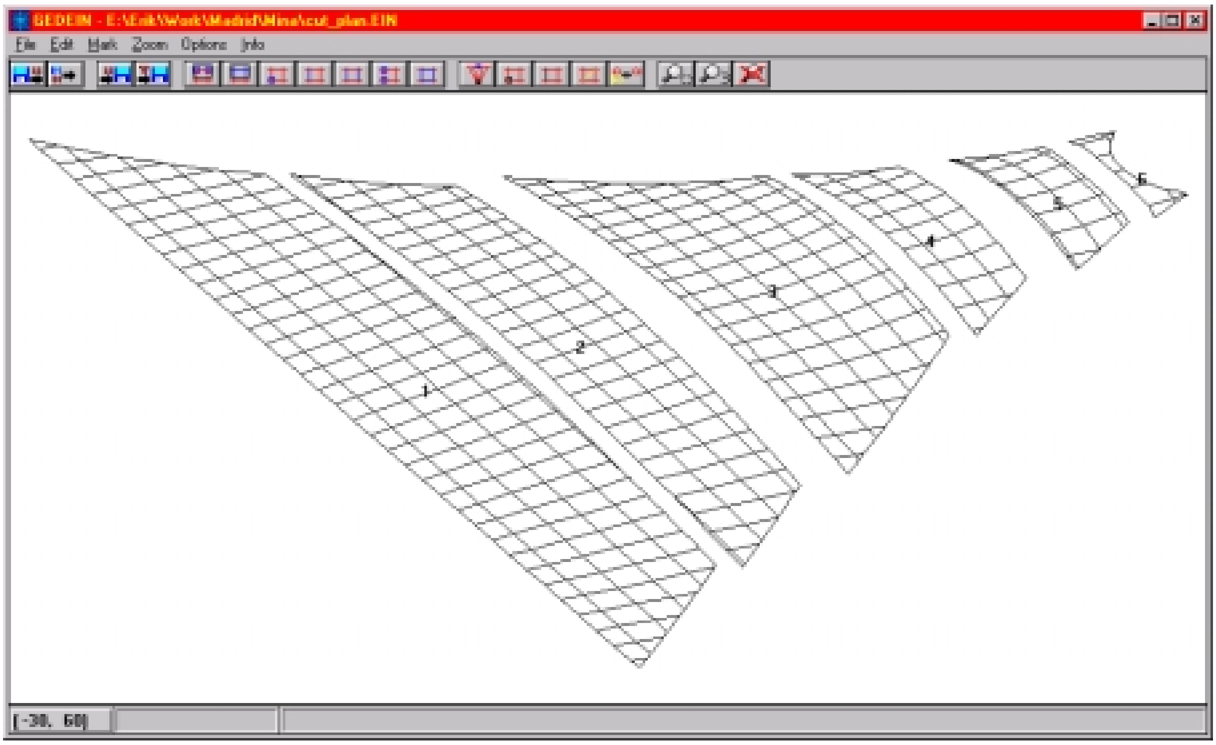

Figure 11: Individual doubly-curved half cloth strips cut from form-found surface.

These half strips are still doubly curved and non-developable. Each sub-surface was then flattened to create planar boundaries. Within each a new regular surface was generated and then mapped back to the 3D space according to a local parametric coordinate system. These new surfaces were then perturbed using a sophisticated adjustment procedure which ensures developability while minimising deviation from the doubly curved sub-surfaces as described above. These developable surfaces were then trimmed using cutting surfaces projected from the doubly curved cloth boundaries using the surface normals and planar half strips resulted. Since the adjacent seam lengths will necessarily now be different, the planar geometry must be adjusted again to ensure compatibility. After joining the half cloths together the patterns, shown in Fig. 12, result. The final procedure to be performed was the cloth compensation and detailing.

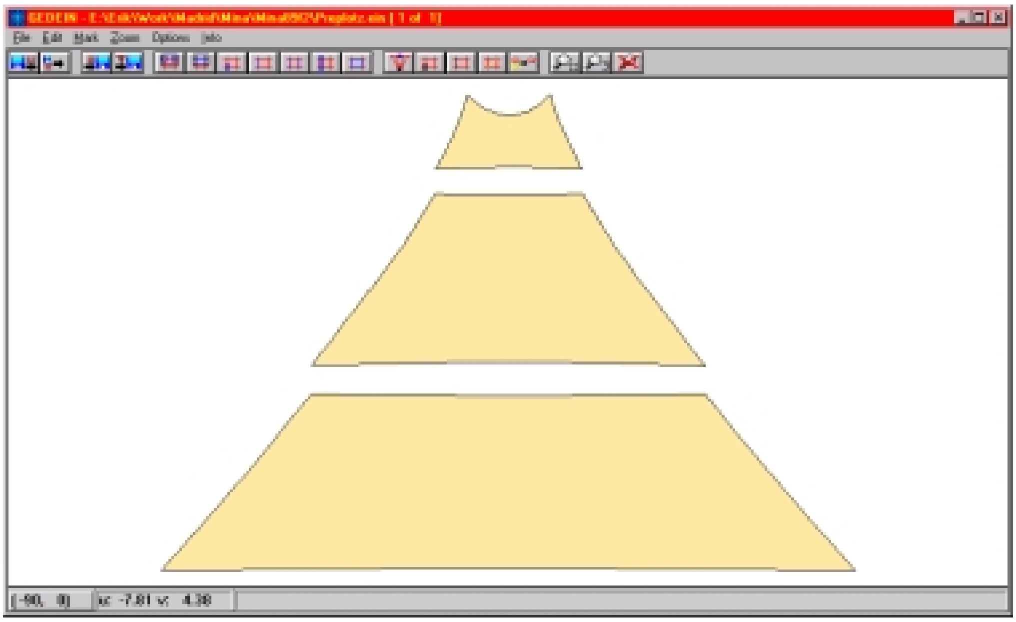

Figure 12: Planar full cloth patterns.

4 Conclusions

It has been shown that, by using a modular approach for the automated cutting pattern generation of architectural membrane structures, the resulting system is extremely powerful and flexible. The very large number of structures which have been successfully designed and patterned using **Easy** prove the validity of this strategy. It is also applicable to other membrane design fields as diverse as clothing and sails. The flexible line generation capability of geodesic and semi-geodesic seam lines is extremely comprehensive, and capable of dealing with problems of much greater complexity than conventional architectural membranes.

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