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FAST MODEL GENERATION AND STATIC CALCULATION OF COMBINED PNEUMATIC AND MECHANICALLY STRESSED STRUCTURES

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1 INTRODUCTION

Today, computer models play an important role in the calculation of textile membrane and foil structures. To be able to derive high-quality results from a model, the software used must enable the most accurate and complete description of a structure. In the case of combined pneumatically and mechanically tensioned structures, the generation of the models and the statical calculation is often a challenge.

Topologically correct discrete mechanical models are a basic prerequisite for static calculations. If, based on the models, additional cutting patterns or the water flow are to be derived, the water tightness must also be guaranteed. Due to the non-linearity of the static calculation, the models must also be in or very close to the state of equilibrium. Only then the geometrically non-linear static problem can be solved.

The state of equilibrium for mechanically stressed membrane and foil structures can only be found with a form-finding calculation. Such structures cannot be designed as conventional structures: conventional design means, in this context, the architect fixes the real geometry on a drawing board. This is not possible with respect to pre-stressed lightweight structures because internal forces or stresses and the surface geometry are not independent of each other [1].

With pure pneumatic structures we are basically dealing with 2 groups. The first group includes the pneumatic structures over an arbitrary boundary. This group needs a form-finding process considering an internal pressure. The second group includes pneumatically feasible structures like cylinders and spherical segments. These shapes can be created by a purely geometric function and in certain circumstances (see 3.2), it is possible to generate the models from group 2 without a form finding process. The direct use (e.g. for cutting pattern generation) of the shapes as a result of geometric functions is often applied when the manufacturing process is to be kept simple and the production costs low.

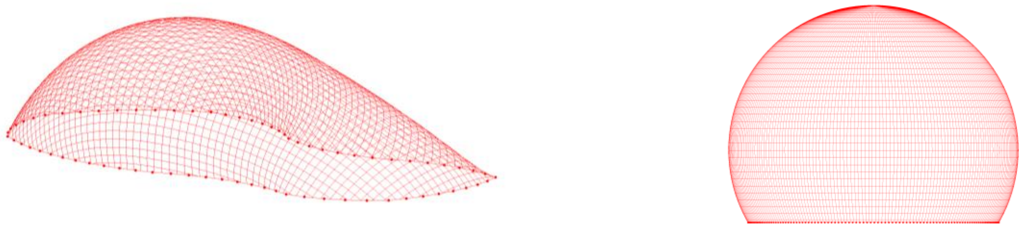


Figure 1: Pneumatic structure over an arbitrary boundary (left), pneumatically feasible structures (right)

Because the form-finding process can be omitted for the second group, it has proven useful to use nurbs geometry to create these structures. Another big advantage of nurbs surfaces is that it is relatively easy to intersect them to create the most complicated overall models.

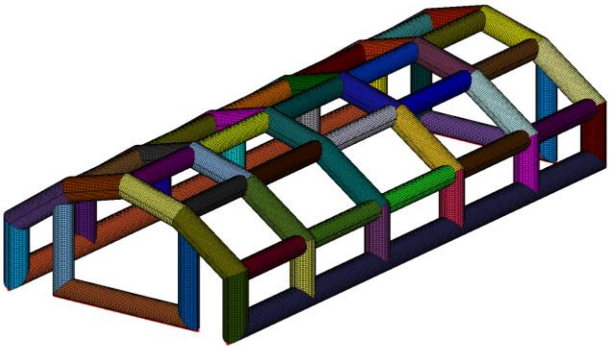


Figure 2: Discrete pneumatically feasible tube structure (1 chamber) made from nurbs

When dealing with combined pneumatically (from group 2) and mechanically stressed structures, the mechanically stressed parts can no longer be derived directly (without form finding) from their nurbs geometry. In this case, form finding is done separately for the mechanically stressed surfaces. The result is then combined with the pneumatically stressed structures to form an overall structure. Our system offers a plugin for the Rhino CAD system for this purpose. The plugin enables the transfer of a nurbs geometry created in Rhino CAD system into the Easy system, i.e. the discrete model is created from intersected Rhino Brep objects.

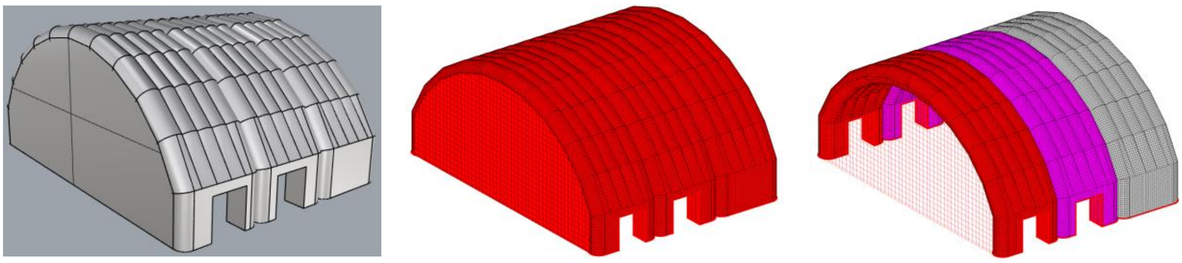


Figure 3: Combined pneumatically and mechanically stressed structure with nurbs (left), discrete overall system (middle), visualisation of the 3 chambers in the discrete model (right)

Statics starts with the definition of the material properties. In general, we define for textile membranes: warp- and weft stiffness, and, if available, crimp- and shear stiffness. We must fix the internal (operating) pressure values for the chamber(s) and now the unstressed geometry of the finite membrane elements for the pneumatic and mechanically stressed elements can be calculated. Load case calculations can be performed now by 4 different modes: Constant inner pressure (p=constant), constant volume (V=constant), constant product of inner pressure and volume (p·V=constant, gas law of Boyle-Mariotte) or even the general gas equation (p·V/T=constant).

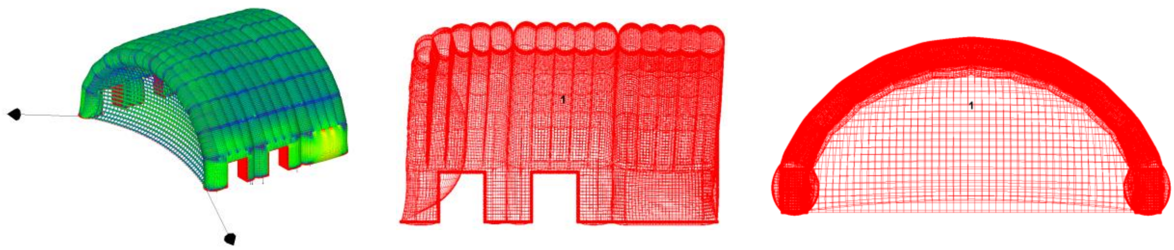


Figure 4: Load case calculation (stresses/reaction forces and deformations)

2 MODEL GENERATION

In the case of mechanically stressed structures, a form-finding calculation is required because internal forces or stresses and the surface geometry are not independent of each other.

As already mentioned in the previous section, in the case of purely pneumatic structures we distinguish between 2 different groups of design principles in model generation. The first group includes structures whose form can only be found via form-finding involving the internal pressure. The second group includes structures that can be formed pneumatically and can be generated via purely geometric functions. Under certain circumstances (see 3.2) a form-finding calculation may be omitted for models in this group. In both cases, a shape should be achieved with harmonic stress distribution.

2.1 Formfinding

The theory of the form finding of mechanically and pneumatically membrane or foil structures has its basics in the well-known Force-Density Method ([1], [4], [5]). By specifying force densities (ratio between force S and stressed length l), the non-linear equilibrium equations become linear and can be solved without specifying initial values.

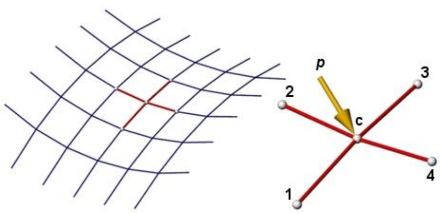


Figure 5: Four cables in point C

In the case of a point C connected by 4 cables to fixed points 1,2,3 and 4, the equilibrium conditions are as follows, where the external load vector can be expressed $\boldsymbol{p}^t = (p_x \quad p_y \quad p_z)$.

$$\begin{aligned}
(x_c - x_1)\frac{S_1}{l_1} + \quad (x_c - x_2)\frac{S_2}{l_2} + \quad (x_c - x_3)\frac{S_3}{l_3} + \quad (x_c - x_4)\frac{S_4}{l_4} &= p_x \\
(y_c - y_1)\frac{S_1}{l_1} + \quad (y_c - y_2)\frac{S_2}{l_2} + \quad (y_c - y_3)\frac{S_3}{l_3} + \quad (y_c - y_4)\frac{S_4}{l_4} &= p_y \\
(z_c - z_1)\frac{S_1}{l_1} + \quad (z_c - z_2)\frac{S_2}{l_2} + \quad (z_c - z_3)\frac{S_3}{l_3} + \quad (z_c - z_4)\frac{S_4}{l_4} &= p_z
\end{aligned} \tag{1}$$

If one specifies known force densities in (1), e.g. $q_1 = \frac{S_1}{l_1}$, and analogue for q_2, q_3 and q_4, then the equations become linear and result in:

$$\begin{aligned}
(x_c - x_1)q_1 + \quad (x_c - x_2)q_2 + \quad (x_c - x_3)q_3 + \quad (x_c - x_4)q_4 &= p_x \\
(y_c - y_1)q_1 + \quad (y_c - y_2)q_2 + \quad (y_c - y_3)q_3 + \quad (y_c - y_4)q_4 &= p_y \\
(z_c - z_1)q_1 + \quad (z_c - z_2)q_2 + \quad (z_c - z_3)q_3 + \quad (z_c - z_4)q_4 &= p_z
\end{aligned} \tag{2}$$

The coordinates of the point $C\ (x_c, y_c, z_c)$ are the solution of these linear equations. In the following step we want to write the system above by considering m neighbours in the point *C*:

$$\begin{aligned}
\sum_{i=1}^{m}(x_i - x_c)q_i - p_x &= 0 \\
\sum_{i=1}^{m}(y_i - y_c)q_i - p_y &= 0 \\
\sum_{i=1}^{m}(z_i - z_c)q_i - p_z &= 0
\end{aligned} \tag{3}$$

The energy which belongs to the system (1) can be written as:

$$\Pi = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) \Rightarrow stat. \tag{4}$$

The internal energy is the expression $\frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v}$. The vector $\boldsymbol{v}^t = (v_x \quad v_y \quad v_z)$ and the matrix $\boldsymbol{R} = diag(q_i \quad q_i \quad q_i)$ show this energy with respect to a single line element i .We can write the inner energy as $\frac{1}{2}q_i(v_x^2 + v_y^2 + v_z^2)$, precisely:

$$\begin{aligned}
v_x &= x_i - x_c \\
v_y &= y_i - y_c \\
v_z &= z_i - z_c
\end{aligned} \qquad \boldsymbol{R} = \begin{bmatrix} q_i & 0 & 0 \\ & q_i & 0 \\ sym. & & q_i \end{bmatrix} \tag{5}$$

The chamber of a pneumatic structure has a volume *V*, which is made by an internal pressure p_i. The product from internal pressure and volume is a part of the total energy Π: a given volume V_0 leads directly to a specific internal pressure p_i: hence the total energy for the form-finding of a pneumatic chamber is

$$\Pi = \frac{1}{2}\boldsymbol{v}^t\boldsymbol{R}\boldsymbol{v} - p_x(x - x_0) - p_y(y - y_0) - p_z(z - z_0) - p_i(V - V_0) \Rightarrow stat. \tag{6}$$

The derivation of the total energy to the unknown coordinates and to the unknown internal pressure ends up with

$$\begin{aligned}
\frac{\partial\Pi}{\partial x} &= \sum_{i=1}^{m}(x_i - x_c)q_i - p_x - p_i\frac{\partial V}{\partial x} &= 0\\
\frac{\partial\Pi}{\partial y} &= \sum_{i=1}^{m}(y_i - y_c)q_i - p_y - p_i\frac{\partial V}{\partial y} &= 0\\
\frac{\partial\Pi}{\partial z} &= \sum_{i=1}^{m}(z_i - z_c)q_i - p_z - p_i\frac{\partial V}{\partial z} &= 0\\
\frac{\partial\Pi}{\partial p_i} &= V - V_0 &= 0
\end{aligned} \tag{7}$$

In the system (7) the internal pressure p_i can be seen as a so-called Lagrange multiplier. The fourth column in (7) shows, that our boundary condition $V = V_0$ is obtained by the derivation of the energy to this Lagrange multiplier. The vector $(\frac{\partial V}{\partial x} \; \frac{\partial V}{\partial y} \; \frac{\partial V}{\partial z})$ describes the normal direction in the point (x, y, z) and the size is the according area. By a set of given force-densities for all elements and a given volume V_0 we end up with a pre-stressed and of course balanced pneumatic system with a volume V_0 and an internal pressure pi.

2.2 Forms from geometrical functions

When using purely geometric functions to generate pneumatic models, it must be ensured that the resulting shapes can be formed pneumatically. If the models are not pneumatically formable, the non-linear static calculation may not lead to any result or the result geometry of the static calculation without external loads may be very far from the originally desired geometry. Only a few geometrical functions are useful as spheres, cylinders, torus shaped forms and segments of these forms.

To keep the manufacturing process simple and production costs low, it is still often seen today that the form-finding process is omitted. Pneumatically feasible forms, such as cylinders and spherical segments are combined to complete structures e.g. for air halls. In the case of cylinders, these are also developable surfaces, which have the advantage that the cutting patterns can be used in straight lines and without distortions in a simple flattening process.

The generation of discrete models in the case of simple, non-intersected structures can be done directly via geometric functions. A discrete spherical model with its centre in the coordinate origin can, for example, be generated with the help of spherical coordinates through

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \rho \cos\theta \sin\phi \\ \rho \sin\theta \sin\phi \\ \rho \cos\phi \end{bmatrix} \tag{8}$$

θ is an azimuthal coordinate running from 0 to 2π, ϕ is a polar coordinate running from 0 to π, and ρ is the radius.

The following relationships apply to a cylinder:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \cos u \\ a \sin u \\ v \end{bmatrix} \tag{9}$$

a is the radius, u is an azimuthal coordinate running from 0 to 2π, v runs from 0 to the desired height. Similarly, simple non-intersected shapes can be generated and combined to form an overall structure.

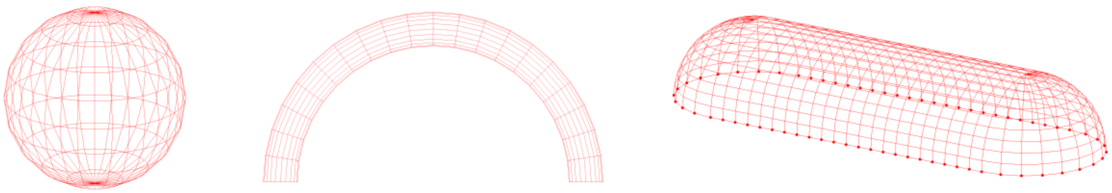


Figure 6: Discrete sphere and tube model (left), 2 quarter spheres and 1 half cylindeer combined to a discrete model (right)

If the structures are combinations based on intersections of the individual basic elements, it is no longer possible to generate the model as simply as shown above. The intersection of discrete structural elements of arbitrary shape is very complex and often leads to unclean topologically incorrect mechanical models. In this case, it has proven useful to build the models based on intersected nurbs surfaces and to derive the discrete mechanical model from the intersected nurbs surfaces.

In our first example, we use the Rhino plugin Brep2Easy to generate the discrete model. The plugin works based on Boundary Representation objects (Brep) objects. Brep objects in turn consist of one or more Brep-face objects. A Brep-face represents the underlying (nurbs) surface including the trimming curves. The plugin generates the discrete mesh within the CAD program, displays it and creates the model data for the Easy system. The user has the possibility to control the material direction and the mesh size for each Brep-face object individually in the CAD system. This creates meshes that represent the material directions. In addition, the discretisation of the edge geometry can be defined independently of the mesh size. By using the Brep objects, it is possible to create topologically clean and watertight discrete models at the intersection lines.

For the static calculation of pneumatic structures, a description of the pneumatic chambers is required in addition to the surface elements describing the area for calculating the external loads (triangular elements). If the Brep object in the CAD system is a closed surface, Brep2Easy automatically provides a discrete chamber description for the Brep object.

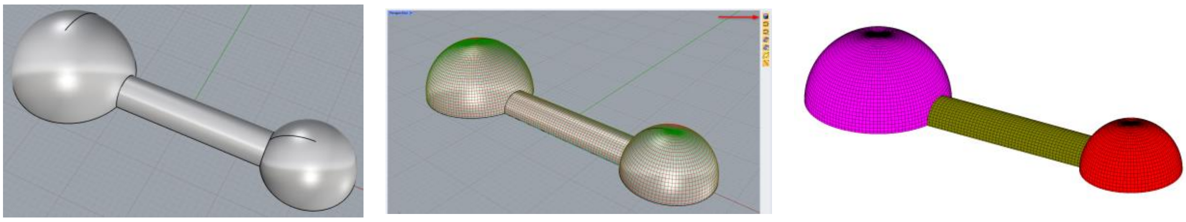


Figure 7: Combination of pneumatically feasible elements by intersecting of nurbs surfaces (left), Brep-object and overlayed discrete model (middle), discrete model (right)

2.3 Combined forms

Our next example shows the procedure for a structure consisting of 2 pneumatically formable tubes and additional 4 mechanically tensioned membrane walls. The CAD model was created through geometric construction and cutting in the CAD System.

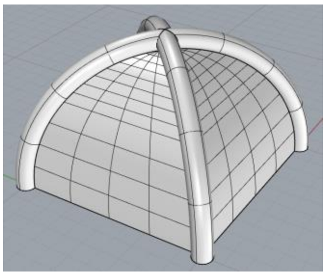


Figure 8: 2 intersecting air pipes and 4 mechanically stressed walls

The two intersecting tubes form one pneumatic chamber and are connected to the 4 membrane walls. The tubes are pneumatically feasible, so there is no need for form-finding here. The 4 walls are mechanically stressed and generated in the CAD system purely geometrically from nurbs surfaces. Form-finding is required for these surfaces.

Because the nurbs membrane walls are useless for correct modelling, only the tubes were generated as discrete surfaces when creating the discrete model. For the 4 wall surfaces, only the boundaries were generated as boundary polygons.

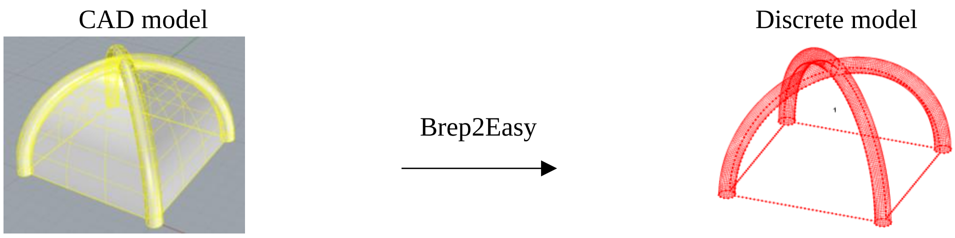


Figure 9: First step in modelling

Based on the boundary polygons, the wall surfaces can then be generated as equilibrium figures with a standard form-finding calculation and combined with the tubes.

Figure 10: Force density form-finding based on boundaries from CAD

The following graphic shows the entire process of model generation. The pneumatically formable tube system with the chamber definition is combined with the form-finding result of the membrane walls. Because the model was derived from a Brep object, a topologically clean mechanical model is created.

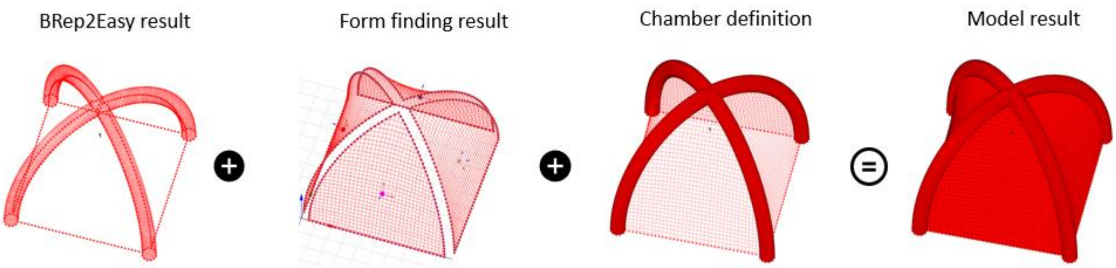


Figure 11: Sequence of the modelling with combined mechanical and pneumatic stressed structures

3 STATICS

A static calculation for membranes and foil structures is geometrically nonlinear. The calculation requires the unstressed geometry and the material properties for all elements of the model. For the load case calculation, the external loads and, in the case of pneumatic structures, internal pressures or volume data are also required. Additional boundary conditions in case of pneumatic structures are that the loads are deformation-dependent and that the gas law must be considered in certain load cases.

3.1 Statics with form-finding models

After the form has been found, the transition from the "force density controlled" calculation to the "elastic force controlled" calculation takes place by introducing the material properties and calculating the undeformed geometry based on the given pre-stress values. After this calculation step, unstressed lengths are available for all elements. The result of this calculation must be identical with the Formfinding result as we 'shortened' the membrane elements in this way. Usually the first load-case in statics to be calculated should be 'internal operation pressure'.

3.2 Statics with geometrically defined models

If a shape has been found purely geometrically, no prestress values are available or the given prestress values usually do not lead to an equilibrium figure. In this case, we suggest the following procedure:

First, the material properties are set for all elements. Because it is not possible to calculate the undeformed geometry due to missing prestress values, the stressed lengths are now set equal to the unstressed lengths. After an initial static calculation with the internal operating pressure, this leads to a geometry change that can be accepted if it is small. If the geometrically defined form was pneumatic feasible the differences are only caused by the elastic deformations in the load case ‘operating pressure’. The following pictures show a shape that cannot be formed pneumatically.

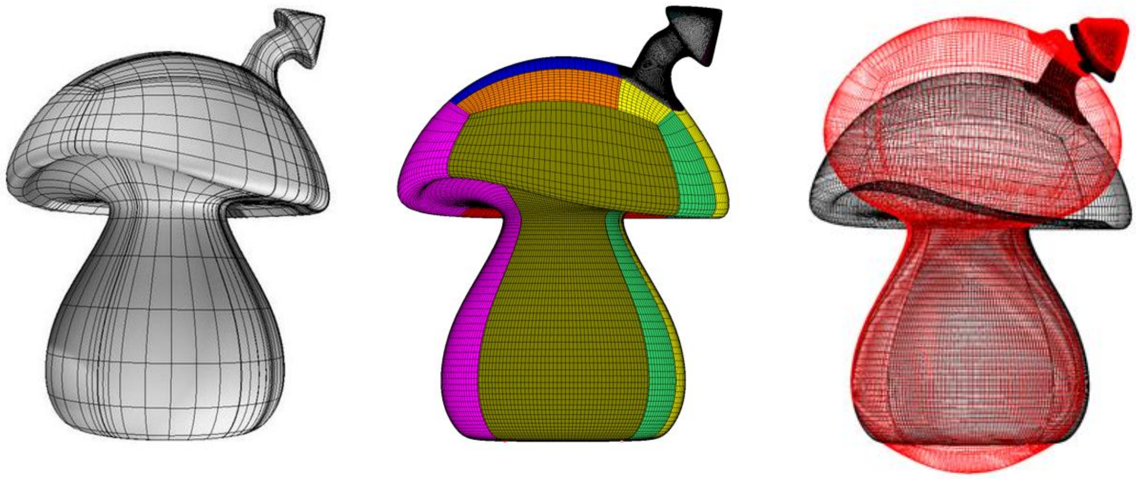


Figure 12: Geometrically defined model (left), discrete model (middle), comparison of the geometrically defined shape (black) with the shape statically calculated under internal pressure (red) (right).

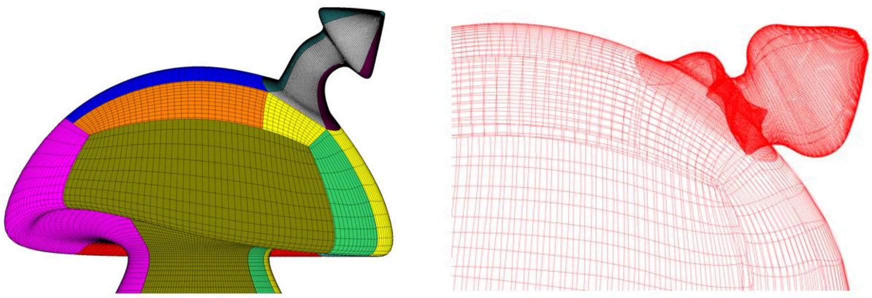
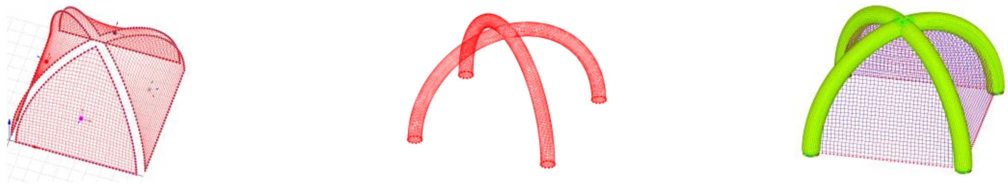


Figure 13: Detailed views: Geometrically defined shape (left), statically calculated shape under internal pressure (right)

3.3 Statics with combined models

When dealing with models where one part was created from a form-finding calculation and another part purely geometrically, one can combine the procedures described in 3.1and 3.2. The material properties are set, then the unstressed element geometry is calculated or set as described above, finally a first load case is calculated with a given internal operating pressure.



Formfinding result. Unstressed length based on prestress | Geometrical defined forms. Stressed length = unstressed length | Load case 'operating pressure'

Figure 14: Combination and calculation of the 'operating pressure' load case

3.4 Theoretical background

By introducing the constitutive equations for the membrane elements into the system (1), we extend the form-finding theory. Now the force-densities q from the form-finding are unknowns and they belong to the material equations.

$$\begin{bmatrix}\sigma_u \\ \sigma_u \\ \tau\end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 0 \\ & m_{22} & 0 \\ sym. & & m_{33}\end{bmatrix}\begin{bmatrix}\varepsilon_u \\ \varepsilon_v \\ \Delta\gamma\end{bmatrix} \tag{10}$$

We must consider that the membrane axial-stress in *u*- or *v*- direction can be expressed as $\sigma_u = \frac{S_u}{b_u}$ and $\sigma_v = \frac{S_v}{b_v}$. b_u and b_v are the widths of the *u*- and *v*-lines. The force-densities q can be introduced now as: $S_u = q_u l_u$ and $S_v = q_v l_v$. The strains in *u*- and *v*-direction can be written as follows: $\varepsilon_u = \frac{l_u - l_{u0}}{l_{u0}}$ and $\varepsilon_v = \frac{l_v - l_{v0}}{l_{v0}}$. The angle difference $\Delta\gamma = \gamma - \gamma_0$ is needed for the shear-stress calculation. γ is the angle between *u* and *v*-direction; $\gamma_0$refers to the 'non-deformed start-situation' without any shear-stress. The geometrical compatibility must be considered as follows: $l_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2}$ and $\gamma = arccos(\frac{l_u * l_v}{l_u l_v})$, in which $(l_u * l_v)$ means the inner (scalar-) product between *u* and *v*-direction. The shear-stress calculation is guaranteed also for a continuous membrane by the fact that the shear angle is between the non-deformed *u*- and *v*-direction of the material [4].

As already mentioned, additional boundary conditions must be fulfilled for pneumatic structures:

1. The internal pressure loads are deformation dependent.
   These loads are non-conservative. To get correct results software packages should consider these effects, especially also for wind loads.
2. Gas laws must be considered in certain load cases. For static calculations we recommend 4 calculation modes:
   a) Given internal pressure p (snow)
   b) Given volume V (water)
   c) Given product $p{\cdot}V$ (Boyle-Mariotte, for example wind, p as absolute pressure)
   d) Given product $\frac{p{\cdot}V}{T}$ (General gas equation, consideration of temperature, p as absolute pressure)

Mode c (consideration of gas-laws) enables the realistic behaviour of the internal pressure. This mode is important in case of e.g. fast wind gusts. Here the pump systems cannot update the inner pressure in the short time. We can see it as a closed system and by considering the temperature as constant we get the gas law of Boyle and Mariotte $p{\cdot}V{=}const$ in this case. Only if the gas law is fulfilled the membrane stresses get the correct size.

$$\begin{aligned}
\frac{\partial \Pi}{\partial x} &= \frac{1}{2}\frac{\partial (v^t R v)}{\partial x} - p_x - \frac{\partial V}{\partial x} p_i &= 0 \\
\frac{\partial \Pi}{\partial y} &= \frac{1}{2}\frac{\partial (v^t R v)}{\partial y} - p_y - \frac{\partial V}{\partial y} p_i &= 0 \\
\frac{\partial \Pi}{\partial z} &= \frac{1}{2}\frac{\partial (v^t R v)}{\partial z} - p_z - \frac{\partial V}{\partial z} p_i &= 0 \\
\frac{\partial \Pi}{\partial p_i} &= V - V_0 &= 0
\end{aligned} \tag{11}$$

Equation (11) refers to mode c, here the constant value $(p_{abs}{\cdot}V)_0$ is the given product and row 4 of (11) must be fulfilled in iterations where the unknown internal pressure pi is adapted.

3.5 Load cases

For the combined model we calculated different wind load cases. The wind loads were generated with the digital wind tunnel EasyDWT (wind speed 30 m/s). As an example, we briefly present the results of the ‘wind +x’ load case calculation. In the example shown, a constant internal pressure of 0.3 bar (30 kN/m^2) was assumed.

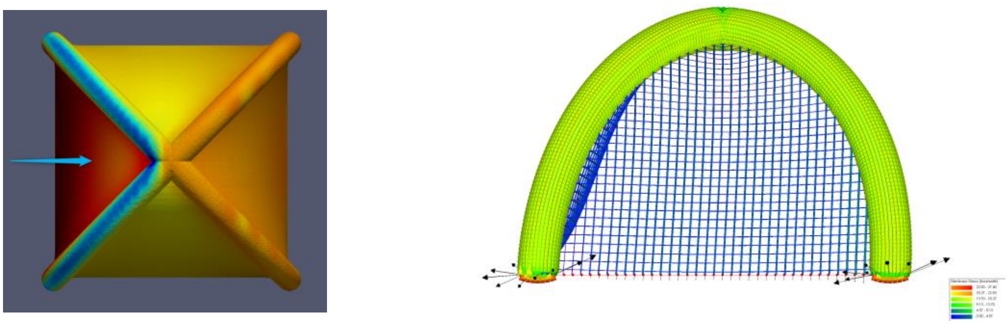


Figure 15: Wind pressures from the digital wind tunnel (left), stresses, reaction forces and deformation (right)

If the load case is calculated with constant internal pressure, it must be assumed that the internal pressure is kept constant at 0.3 bar during the wind load.

For the load case ‘wind +x’, calculated with the gas law, the result was not an increase in volume with a simultaneous decrease in internal pressure, as initially expected, but a reduction in volume with a simultaneous increase in internal pressure (from 30 kN/m^2 to 30.11 kN/m^2).

4 Summary

The modelling for a static calculation is still a challenging and time-consuming task today, especially when it comes to intersected pneumatic structures. In our article we have shown that the fast generation and static calculation of combined pneumatically and mechanically tensioned models is possible with our system. A particular difficulty in model generation is the intersection of individual volume elements. To avoid the intersection problems in the case of discrete meshes, we use Nurbs surfaces and intersect them. The newly created partial surfaces are combined as objects and then discretised. This results in topologically correct mechanical models that are suitable for a static calculation.

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